The Digital Economist

Lecture 7 – Producer Behavior

A PRODUCER OPTIMUM

A **producer optimum** represents a solution to a problem facing all business firms -- maximizing the profits from the production and sales of goods and services subject to the constraint of market prices, technology and market size. This problem can be described as follows:

$$\max \pi = P_x(X) - [wL + rK + nM + aR]$$

s.t

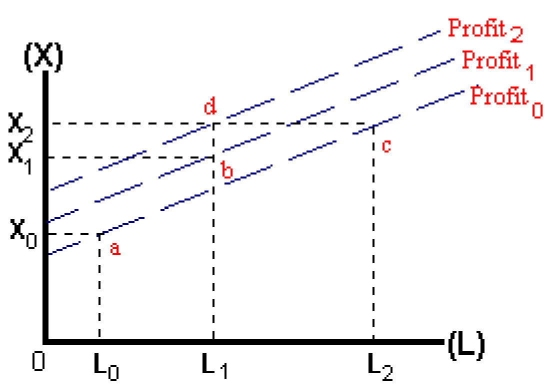
$$X = f(L,K,M,R).$$

In this *optimization problem*, the profit equation represents the objective function and the production function represents the constraint. The firm must determine the appropriate input-output combination as defined by this constraint in the attempt to maximize profits. The objective function can be rewritten in the form of 'X = f(L)' as follows:

$$X = [(\pi + \mathbf{FC})/P] + (w/P_x)L$$

where **FC** represents the fixed costs of production (rK + nM + aR). This expression is known as an **iso-profit** line with the term in the brackets being the intercept which represents a given level of profits and the term (w/P_x)-- also known as the real wage rate, represents the slope of this line. Any point on a particular line represents a given level of profits.

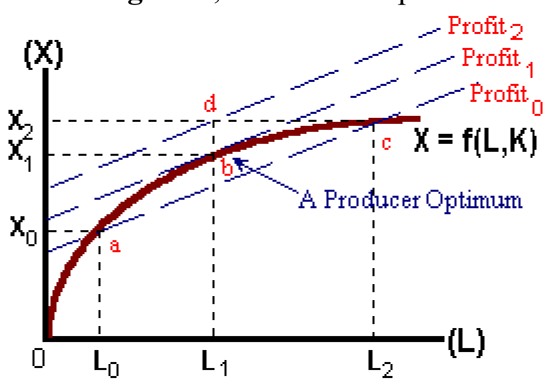
Figure 1, Lines of Equal Profits



The combination of L_0, X_0 corresponds to a level of profits of π_0. Likewise the combination of L_2 (*greater costs*) and X_2 (*more revenue*) also corresponds to this same level of profits (π_0) -- *revenue and costs increase by the same amount*. However, the combination of L_1 and X_1 correspond to a greater level of profits relative to the combination of L_0, X_0 (*revenue increases more than costs*). If we compare the input-output combination of point '**d**', (L_1, X_2), to the combination at point '**b**' (L_1, X_1), we find that profits have increased even further given that more output is being produced (and thus more revenue generated) with the same amount of labor input. Comparing the production combination at point '**c**', (L_2, X_1), to the combination at point '**b**' (L_1, X_1), we find that profits decline since we are using more labor to produce the same level of output.

By adding the production function to the above diagram, we find that the input- output combinations as defined by points '**a**', '**b**', and '**c**' are all within the limits of available technology. Point '**d**' however, is *unattainable* -- a level of output of X_2 is impossible with a level of labor input of L_1.

Figure 2, a Producer Optimum



At point '**b**', we find that we achieve the greatest level of profits possible with this existing level of technology. At this point the production function is just tangent to iso-profit line 'Profit_1'. This point is known as a **producer optimum**. The condition for this optimum is formally defined as:

slope of an iso-profit line = slope of the production function

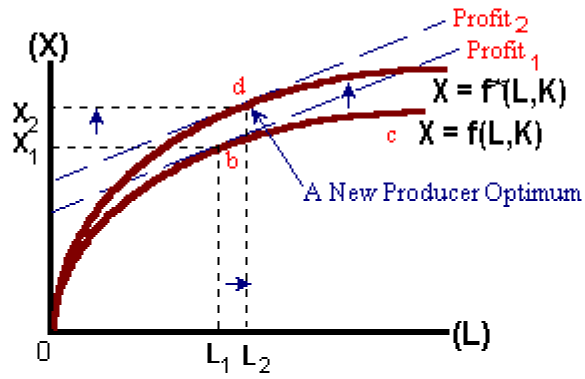
or

$$(w/P_x) = MP_{Labor}$$

External Shocks

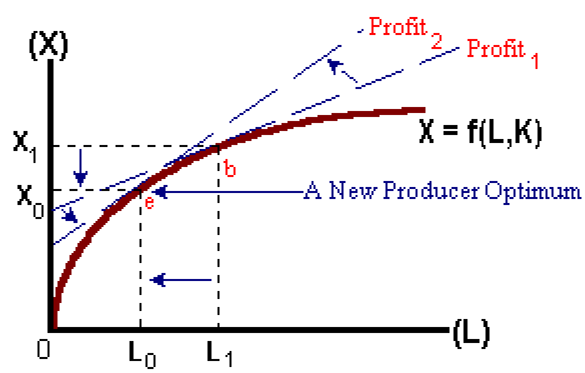
An increase in labor productivity (either due to better technology or the availability of more capital) will shift the production function upward. The firm will hire more labor (if possible at existing wage rates), produce more output for sale and (assuming that output prices remain the same) achieve a greater level of profits. This shock in shown in figure 3 below:

Figure 3, an Increase in Labor Productivity



In the case of an increase in the wage rate, we find that the slope of any iso-profit line becomes steeper and thus tangent to the production function at some point to the left of the original. At this new producer optimum, we find that the firm will react by hiring less labor now that this input is more expensive, and as a consequence reduces the level of output produced. In this example, revenue falls, and the costs of production increase (less labor but at a higher wage rate). The profits of the firm will be reduced.

Figure 4, an Increase in the Wage Rate



COSTS and COST RELATIONSHIPS in the SHORT RUN

A prelude to understanding the costs of production in the short run is a discussion of the **stages of production**. These stages represent different relationships between the quantities of the variable factor input used (typically labor) and the quantities of the fixed factors of production available.

The Stages of Production:
Stage I exists where $MP_L > AP_L$ that is, where using more labor (the variable factor of production) leads to more output (X) and more effective use of the fixed factors or production. This is evidenced by increases in Average Productivity (AP_L). If the margin is greater than the average, *the margin is "pulling" the average up.*

Stage II exists where $AP_L > MP_L > 0$. In this stage, increasing the amount of labor used leads to additional output although output per worker (AP_L) is declining. If the margin is less than the average, then the margin is pulling the average down.

Stage III is where the Marginal Productivity of Labor is negative -- additional labor input results in less output (negative returns). In this stage of production, there is too much of the variable input relative to the amounts of fixed factors of production available.

Thus in **Stage I** there is *too much of the fixed factors of production relative to the variable factors of production* and the firm should increase production. In **Stage III** the opposite is true (*too much of the variable factor relative to the fixed factors*) and the firm should reduce the level of production (by using less of the variable factor -- labor). **Stage II** represents a balance between the fixed and variable factors of production and the firm should produce in this range. The exact amount of labor to be used would be determined by the condition for a producer optimum:

$$MP_L = w/P_X$$

The Costs of Production

A production function describes the underlying technology that governs the conversion of inputs into the desired output. By simply pre-multiplying the quantity of each factor of production by its associated factor price, this production technology can be either modeled by or govern related cost relationships. This relationship is known as the **dual** relationship between production and costs.

These costs can be described as follows:

Total Costs: **TC** = Variable Costs (**VC**) + {Fixed Costs (**FC**)}

Given that

VC = wL

and

FC = {rK + nM + aR}

In per-unit terms:

Average Variable Costs: **AVC** = VC / X
= wL / X
= w(L/X)
= w / AP_L

thus as $AP_L \uparrow$, AVC $\downarrow$, *and vice-versa*.

Average Fixed Costs: **AFC** = FC / X

and as X $\uparrow$, AFC $\downarrow$,

Average Total Costs: **ATC** = TC/X *or* **AVC** + **AFC**

and **Marginal Costs** (the cost of producing one more unit of output):

$$\begin{aligned}\textbf{MC} &= \Delta\text{Total Costs}/\Delta X,\\ &= \Delta\text{Variable Costs}/\Delta X,\\ &= \Delta(wL)/\Delta X,\\ &= w(\Delta L/\Delta X) = w/(\Delta X/\Delta L),\\ \textbf{MC} &= (w/MP_L)\end{aligned}$$

as $MP_L \downarrow$, $MC \uparrow$, *and vice-versa*.

If we accept that the firm will only operate in **Stage II** where $AP_L > MP_L$ then given the dual nature of production and costs we have:

MC > AVC

and additionally:

as $X \uparrow$, $MC \uparrow$.

This segment of Marginal Costs represents the **Supply curve** for the firm.

Table 1, The Costs of Production
Production Function: $X = 18L^2 - L^3$ -- *Wage Rate* = 25.00

Labor Input (L)	**Output (X)**	**MP_L**	**Fixed Costs**	**Variable Costs**	**Total Costs**	**Avg. Variable Costs**	**Avg. Total Costs**	**Marginal Costs**
0	0	NA	100.00	0.00	50.00	NA	NA	NA
1	17	33	100.00	25	125	1.471	7.353	0.758
2	64	60	100.00	50	150	0.781	2.344	0.417
3	135	81	100.00	75	175	0.556	1.296	0.309
4	224	96	100.00	100	200	0.446	0.893	0.26
5	325	105	100.00	125	225	0.385	0.692	0.238
6	432	108	100.00	150	250	0.347	0.579	0.231
7	539	105	100.00	175	275	0.325	0.51	0.238
8	640	96	100.00	200	300	0.313	0.469	0.26
9	729	81	100.00	225	325	0.309	0.446	0.309
10	800	60	100.00	250	350	0.313	0.438	0.417
11	847	33	100.00	275	375	0.325	0.443	0.758
12	864	0	100.00	300	400	0.347	0.463	0
13	845	-39	100.00	325	425	0.385	0.503 -	0.641
14	784	-84	100.00	350	450	0.446	0.574 -	0.298
15	675	-135	100.00	375	475	0.556	0.704 -	0.185

The Shut-down point
We can rearrange our condition for Producer Optimum:

$$MP_L = w/P_X$$

as:

$$P_X = w/MP_L$$

with the right-hand side term being **Marginal Costs** ($w/MP_L = MC$ – *see above*):

$$P_X = MC$$

The profit-maximizing firm will produce a level of output where market price just covers the marginal cost of production for that level of output. Now suppose that we have the following subset of data:

Table 2, the Shut-down Point

Output (X)	**FC**	**VC**	**TC**	**AVC**	**ATC**	**MC**
10	50	92	142	9.20	14.90	7.00
11	50	100	150	9.10	14.20	8.00
12	50	115	165	9.60	13.80	15.00
13	50	140	190	10.80	14.60	25.00

If:

Price =	**15.00**	**10.00**	**7.00**
Output (X) =	**12**	**11**	**10**
Revenue =	180.00	110.00	70.00
Total Costs =	165.00	150.00	142.00
Profit =	+15.00	-32.00	-72.00

At a market price of 15, the *profit-maximizing* firm will produce a level of output equal to 12 units and earn (abnormal) profits of 15.

At a market price of 10.00, the profit-maximizing firm (*now loss minimizing*) will produce 11 units of output. Even though there are losses of (-32.00), it is still to the advantage of the firm to continue to operate. If the firm were to shut-down, it would still be responsible for its fixed costs of 50.00. So in this case, as long as:

$$ATC > P > AVC$$

Revenue still covers all of the **Variable Costs** of production and makes a contribution against the **Fixed Costs** of production.

At a lower market price of 7.00, the firm would choose to produce 10 units of output. However, the market price does not even cover the per-unit (average) variable costs and thus total losses exceed the fixed costs of the firm. In this case where:

$$P < AVC$$

it is better for the firm to cease operation. Also note that when this is the case,

$$P < AVC \text{ and } P = MC$$

so,

$$MC < AVC$$

or

$$MP_L > AP_L$$

and the firm is trying to operate in **Stage I** of production.

In summary, we can define the relevant supply decisions by the firm, in the short run, as being where:

$$P, MC > AVC$$

and given that this is consistent with Stage II or production:

as $X \uparrow, MC \uparrow,$

As market price (P) increases, the profit maximizing firm will offer more output (X) to the market.

PRODCTION IN THE LONG RUN

Production in the long run is distinguished from short run production in that all factor inputs may be used in varying amounts. Given the production function:

$$X = f(L, K, M, R),$$

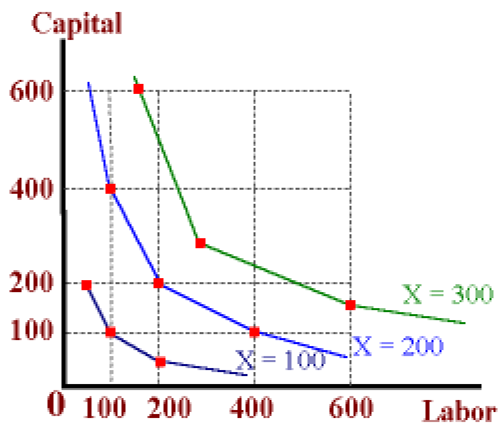
we find that one factor may be substituted, to some degree, for another factor of production. Increasing the amount of capital or machinery 'K' can replace some labor 'L' but not all of the labor in a production process. Increasing amounts of labor (greater care being taken in production to avoid waste) can reduce the need for some material inputs 'M'. In addition, where all factors of production are allowed to vary in quantity, proportional increases in all factors of production may lead to unbounded increases in output.

As we begin to model production in the long run, we will simplify the production function somewhat as:

$$X = f(L, K),$$

where we assume that the extraction of raw materials or the development of land is accomplished with combinations of labor and capital input. Entrepreneurship is embedded in the production technology used [f(.)]. This allows for a two-dimensional representation of combinations of factor inputs required to produce chosen levels of output.

Figure 5, Factor Input Combinations



Suppose, for example, it is possible to produce 100 units of output (X = 100) with the following combinations of labor and capital :

L	**K**	
50	200 --	*Capital Intensive Production*
100	100 --	*Equal Amounts*
200	50 --	*Labor Intensive Production*

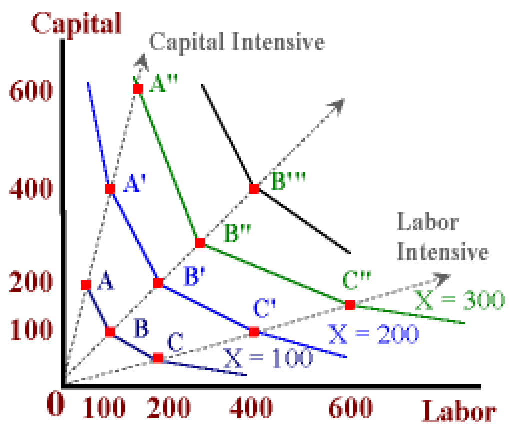
Each point on the navy-blue line in the above diagram represents these input combinations. The lines connecting each point denote the possibility that an arithmetic average of any of these combinations may also allow for the production of 100 units of output.

If the production technology allows, we could double the quantity of each input and perhaps double the amount of output. These points (on the blue lines) represent capital and labor combinations that allow for this greater level of output. By tripling the original quantity of inputs (green lines) might allow for a tripling of output.

The 'kinked' lines in the above diagram are known as Production Isoquants or "*lines of equal output*". Each point on a given colored line represents combinations of the two inputs that allow for a given level of output: **X = 100**, **X = 200**, or **X = 300**.

In the following diagram, points **A**, **A'**, or **A''** represent combinations of capital and labor used in a 4:1 ratio in order to produce the three levels of output. In relative terms, this is known as **Capital Intensive Production**.

Figure 6, Production Isoquants

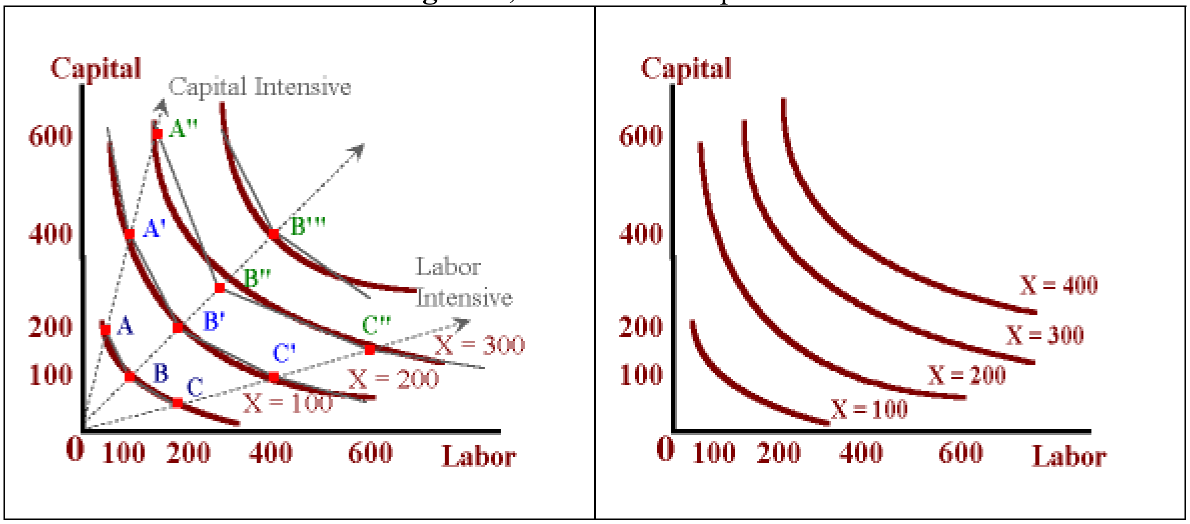


The points **C**, **C'**, or **C''** in this same diagram, represent combinations of capital and labor used in a 1:4 ratio or **Labor Intensive Production**.

For a given production technology it is not possible to say that using one factor more intensively than the other is better or more efficient. In economic systems where *capital is relatively scarce* and therefore relative more expensive in use as compared to labor, a labor intensive production process may be more efficient. If the opposite is true *(labor being relatively scarce*), then capital intensive production may be observed. The actual combination of factor inputs will depend on their relative productivities and existing factor prices.

The three rays representing different production processes (capital intensive, labor intensive, or in-between), may not be the only options available. Allowing for a continuim of processes results in the 'kinked' production isoquants becoming smoother. These smooth isoquants represent an infinite number of production processes available.

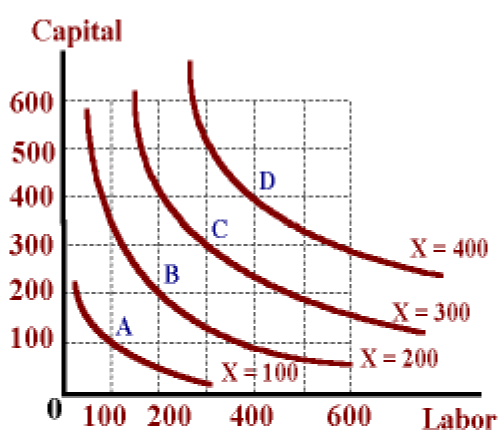
Figure 7, Production Isoquants



Returns to Scale
Through an examination of proportional increases in the inputs, we can define different production technologies with the concept of **returns to scale**. This concept refers of the ability to *more than double, exactly double,* or *less than double* the level of output when the quantity of all the available inputs are exactly doubled.

For example, in some cases, a production process may be replicated. Thus if a certain quantity of grain is being produced on one acre of land with $\mathbf{L}_o$ units of labor input and $\mathbf{K}_o$ pieces of capital, then by replicating this production process the quantity of grain produced may be doubled. In this case, the technology represented is known as **constant returns to scale**.

Figure 8, Constant Returns to Scale



This allows for changes in the amount of labor 'L' and capital 'K' used for different levels of production or output. Note that in order to produce 100 units of output (**X = 100**), 100 units of labor and capital are required. For 200 units of output, 200 units of both labor and capital are required (a 100 unit increase for each factor of production).

Finally, for 300 units of output, 300 units of labor and capital are required. Proportional changes in the quantity of inputs results in proportional changes in output.

Technologies where a doubling of inputs leads to a more than doubling of outputs is known as **increasing returns to scale** (**Figure 9**). Finally production technologies that lead to a less than doubling of output when all inputs are doubled is known as decreasing returns to scale (**Figure 10**).

Figure 9, Increasing Returns to Scale

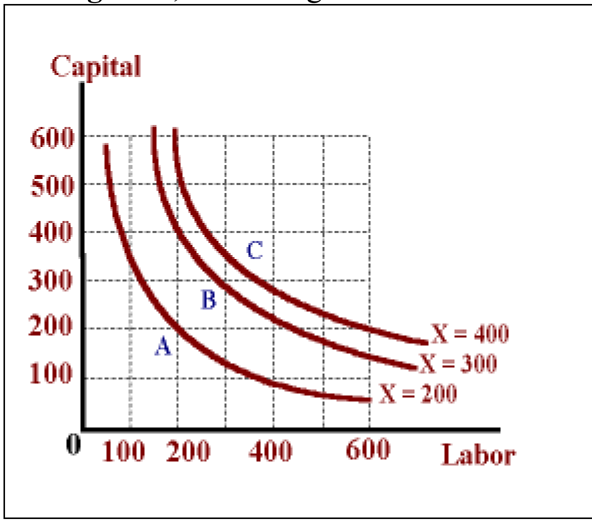
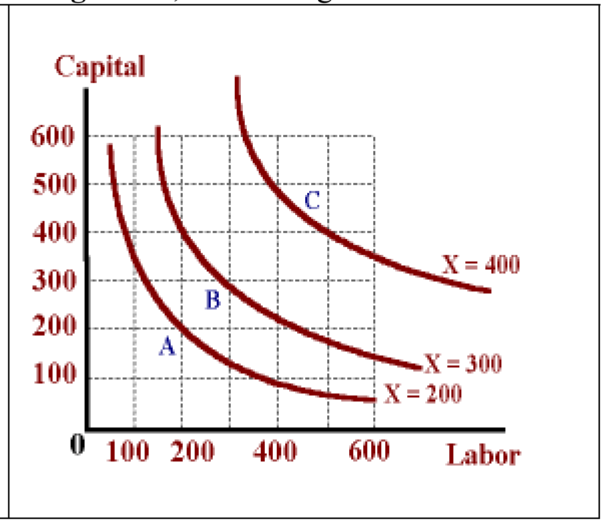


Figure 10, Decreasing Returns to Scale



The Cobb-Douglas Production Function
One mathematical production relationship that possesses three properties for production (*diminishing marginal productivity, essential inputs, and possibilities for substitution*) is the **Cobb-Douglas production function**. This particular representation is one of several mathematical possibilities and may be written as follows:

$$X = \mathbf{A}_t L^{\alpha} K^{\beta}$$

where L and K represent the factor inputs listed above, $\mathbf{A}_t$ represents a measure of technology at time period 't', and the exponents represent production parameters (*actually output elasticities*). The fact that it is multiplicative in the inputs reflects the notion that one factor may be substituted for another. Diminishing marginal productivity requires that the exponents α and β each take on values less than one. Each input being essential and making a positive contribution to output implies that these exponents be strictly greater than zero.

The different production technologies are defined by the sum of the production exponents. Constant returns to scale would imply that α and β sum to one:

Given:

$$X = \mathbf{A}_t L^{\alpha} K^{\beta}$$

If the quantity of both inputs ('L' & 'K') were doubled:

$$\mathbf{A}_t(\mathbf{2L})^{\alpha}(\mathbf{2K})^{\beta} = \mathbf{2}^{(\alpha+\beta)}\mathbf{A}_t L^{\alpha}K^{\beta} = \quad 2^1\,\mathbf{A}_t L^{\alpha}K^{\beta} \quad = \mathbf{2}X$$

With **increasing returns to scale** these exponents will sum to a value greater than one (such that $\mathbf{2}^{(\alpha+\beta)} > \mathbf{2}$) and with decreasing returns to scale, these exponents sum to a value less than one (such that $\mathbf{2}^{(\alpha+\beta)} < \mathbf{2}$).

Returns to scale represent one dimension of production technology in the long run. This concept governs how costs change as production levels are altered. Under constant returns to scale, a doubling of output results in an exact doubling of production costs. In the case of increasing returns, costs increase at a rate less than the change in output such that average (per-unit) costs decrease with increasing levels of output. On the other hand, under decreasing returns to scale, costs increase at a rate greater than production. In this case, increasing production levels are matched by increasing per-unit costs.

Substitution among factor inputs
A second dimension to production technology is the ease by which one factor may be substituted for another. This may be necessary as relative factor prices change (i.e., wages increase such that labor becomes more expensive relative to capital) and the firm attempts to substitute away from the more expensive factor.

> *The Cobb-Douglas production function is just one particular mathematical form that is very restrictive with respect to different degrees of factor substitution.*

Two extreme cases with respect to factor substitution are a **Linear Technology** where the production function may be written as:

$$X = \alpha\mathbf{L} + \beta\mathbf{K}$$

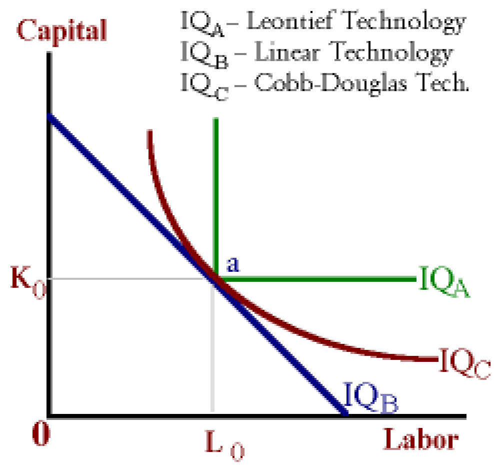
In this case, the factors are *perfect substitutes* for one another and the profit maximizing firm will use only one or the other factor in production. In this case neither factor is essential in the production process.

At the other extreme is a **Leontief Technology** where factors must be used in fixed proportion to one-another (i.e., in providing passenger services, one bus is matched with one driver):

$$X = \mathit{min}[\alpha\mathbf{L}, \beta\mathbf{K}]$$

In this case, *substitution is not possible* and the firm must absorb factor price increases in the form of higher costs.

Figure 11, Elasticity of Substitution



These different expressions may be summarized in a single mathematical form known as the **Constant Elasticity of Substitution** (CES) production function:

$$X = \mathbf{A}[\alpha \mathbf{L}^{\rho} + \beta \mathbf{K}^{\rho}]^{(1/\rho)}$$

The new parameter introduced 'ρ' is a measure of the ease by which labor may be substituted for capital or vice-versa. If the following values of 'ρ' are observed:

- **$\rho = 1$** -- then we have a **Linear technology**,
- as **$\rho \rightarrow 0$** -- then we have a **Cobb-Douglas technology**,
- as **$\rho \rightarrow -\infty$** -- then a **Leontief technology** exists.

Stated differently, the additional parameter 'ρ' is a measure of the convexity of the production isoquants such that as the value of this parameter approaches one, the isoquants become more linear and greater ease in factor substitution exists.

The Marginal Rate of Technical Substitution
Given the following production function:

$$X = f(L, K)$$

we can write (via total differentiation):

$$\Delta X = MP_L \Delta L + MP_K \Delta K,$$

that is, changes in output (*in the long run*) are measured as the sum of changes in labor input (via the *marginal productivity of labor*) and / or changes in capital (via the *marginal productivity of capital*). Holding output constant (**$\Delta X = 0$**, as we would on a given production isoquant), we can derive:

$$\mathbf{0} = MP_L \Delta L + MP_K \Delta K,$$

or

$$\Delta K / \Delta L = -MP_L / MP_K,$$

This last result defines the *slope of a Production Isoquant* '$\Delta K / \Delta L$' as being equal to the ratio of marginal productivities 'MP_L / MP_K'. This ratio is also known as the **Marginal Rate of Technical Substitution** (**MRTS**) which measures the rate by which one factor may be substituted for another.

Using the Cobb-Douglas production function as a particular mathematical function we can derive:

$$X = \mathbf{A}L^{\alpha}K^{\beta}$$

and

$$MP_L = \alpha\mathbf{A}L^{\alpha-1}K^{\beta} = \alpha X/L$$

$$MP_K = \beta\mathbf{A}L_{\alpha}K^{\beta-1} = \beta X/K$$

and the **Marginal Rate of Technical Substitution**:

$$\mathbf{MRTS} = MP_L / MP_K = \alpha K / \beta L.$$

In the case of Cobb-Douglas technologies, the MRTS is proportional to the ratio of factor inputs used.

In comparison, if we examine a linear technology:

$$X = \alpha L + \beta K$$

$$MP_L = \alpha,$$

And

$$MP_K = \beta$$

Such that:

$\mathbf{MRTS} = \alpha/\beta$ that remains constant independent of factor-input ratios.

Profit Maximizing Behavior in the Long Run

Given a profit equation:

$$\pi = P_X[f(L, K)] - (wL + rK)$$

where the term in the square brackets represent output via the production function $[X = f(L,K)]$.

The first-order conditions are:

$$d\pi/dL = P_X[MP_L] - w = 0$$

and

$$d\pi/dK = P_X[MP_K] - r = 0.$$

If we solve for 'P_X' (the market price of the output) in both equations and set them equal to each-other we have:

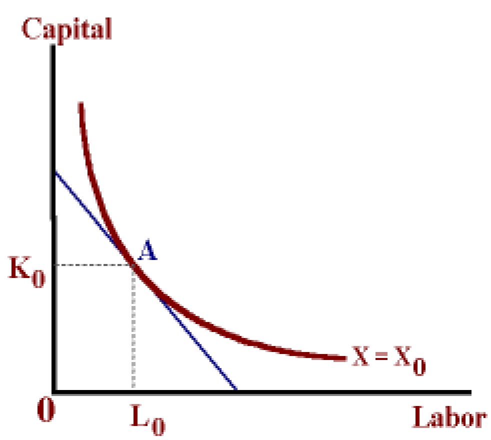
$$MP_L/w = MP_K/r$$

or

$$MP_L / MP_K = w/r$$

The condition for **profit maximization** (*or cost minimization*) is where the **MRTS** is just equal to the ratio of factor-input prices (‘**w**’ & ‘**r**’). This condition is known as a **Producer Optimum in the Long Run** and defined for a given level of output.

Figure 12, A Producer Optimum



Note that in the case of the Cobb-Douglas production function, the Producer Optimum may be defined as:

$$\alpha\mathbf{K} / \beta\mathbf{L} = (w/r)$$

A profit-maximizing combination of these two inputs would be:

$$\mathbf{K} / \mathbf{L} = (\beta/\alpha)\ (w/r)$$

or

$$\mathbf{K} = (\beta/\alpha)\ (w/r)\mathbf{L}.$$

For example if the specific Cobb-Douglas production function is estimated as:

$$X = \mathbf{1000}L^{0.80}K^{0.20}$$

and the **wage rate 'w'** is equal to \$20.00 and **cost per unit of capital 'r'** is equal to \$10.00,

$$\mathbf{K} = (0.2/0.8)(\$20.00/\$10.00)\mathbf{L}$$
$$= (1/4)(2/1)\mathbf{L}$$

or

$$\mathbf{K} = (1/2)\mathbf{L}$$

The firm would use capital and labor in a 1:2 ratio (2 units of labor for each unit of capital). This makes sense since labor is *four-times* as productive as a unit of capital (α=0.80 and β =0.20) but *only twice* as expensive.

In the case of a **linear technology** if

$$MRTS > w / r$$

or

$$\alpha / \beta > w / r$$

the firm would produce using only labor since that factor's productivity relative to its price is greater than that of capital:

$$\alpha / w > \beta / r$$

LONG RUN COSTS

A long run cost equation (*given two factor inputs*) may be written as:

$$C = wL + rK$$

or solving for K = f(L) -- *slope-intercept form*:

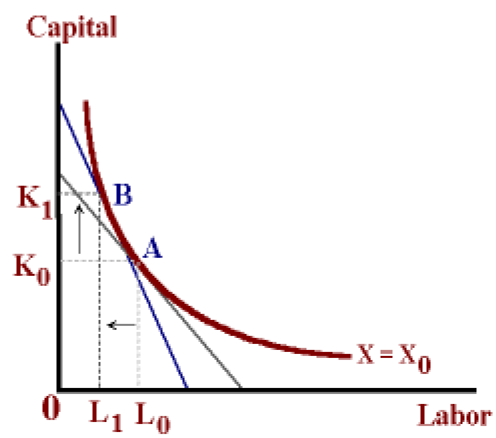
$$K = C_o/r + (w/r)L.$$

This expression is known as the **Iso-Cost line** or *line of equal costs* with a slope defined by the ratio of factor prices (w/r) and shown in the above diagram (**figure 12**) as the **navy-blue** line.

Changes to output levels would require more of both inputs such that costs would increase. As long as the ratio of factor prices does not change, the ratio of factor-intput use will also not change.

An increase in one of the factor prices will lead the profit-maximizing (cost-minimizing) firm to substitute away from the factor that has become more expensive and towards the relatively cheaper factor-input. For example, an increase in the wage rate will lead the firm to find a different combination of inputs in order to produce the same level of output. In this case the firm will substitute away from labor ($L_0 \rightarrow L_1$) and towards capital ($K_0 \rightarrow K_1$) as shown in the diagram below:

Figure 13, An Increase in the Wage Rate



In the case of a **Cobb-Douglas** technology this substitution is possible such that costs at point '**B**' have increased relative to point '**A**' but *by a smaller amount* than if substitution were not possible.

$$\Delta\textbf{Costs} = (\Delta_{[+]}w)[\mathbf{L_1 - L_0}] + (r)[\mathbf{K_1 - K_0}]$$

but

$$(\Delta_{[+]}w)[\mathbf{L_1 - L_0}] + (r)[\mathbf{K_1 - K_0}] < (\Delta_{[+]}w)[\mathbf{L_0}] + (r)[\mathbf{K_0}]$$

This would occur with a **Leontief technology** where factor-inputs must always be used in fixed proportion. In this case the costs of production would increase in direct proportion to the increase in the wage rate:

$$\Delta\textbf{Costs} = (\Delta w)\mathbf{L_0}$$

This helps explain why factor price increases are strongly resisted in industries governed by a **Leontief technology --** the best example being the airlines with respect to labor contract negotiations.

Be sure that you understand the following concepts and definitions:

- Diminishing Marginal Productivity
- Inefficient Production
- Long Run Production
- Marginal Productivity of Labor
- Marginal Rate of Transformation
- Opportunity Cost
- Production Function
- Production Possibilities
- Relative Prices
- Short Run Production
- Technology
- Unattainable Output Combinations
- Iso-Profit Line
- Producer Optimum
- Profit Maximization
- Real Wage
- Average Fixed Cost (AFC)
- Average Productivity (AP)
- Average Total Cost (ATC)
- Average Variable Cost (AVC)
- Costs (of Production)
- Fixed Factors of Production
- Marginal Costs (MC)
- Profit Maximization
- [Sales] Revenue
- Stage I (of Production)
- Stage II (of Production)
- Stage III (of Production)
- Supply Curve (for the firm)
- Total Costs (TC)
- Variable Costs (VC)
- Variable Factor of Production
- Capital Intensive Production
- Cobb-Douglas Production Technology
- Constant Elastiticy of Substitution (CES)
- Constant Returns to Scale
- Decreasing Returns to Scale
- Elasticity of Substitution
- Factor Substitution
- Increasing Returns to Scale
- Labor Intensive Production
- Leontief Production Technology
- Linear Production Technology
- [the] Long Run
- Production Isoquant
- Returns to Scale
- Iso-Cost Line
- Marginal Rate of Technical Substitution (MRTS)

Optimizing Conditions Discussed:

$MP_L = w/P_x$ ⇒ *** A Producer Optimum in the Short Run ***

or

$P_x = w/MP_L$

P_x=MC ⇒ *** Profit Maximiztion in a Competitive Environment ***

MRTS = w/r (**MRTS** *defined as* MP_L / MP_K)

so

MP_L / MP_K = **w/r** ⇒ ***A Producer Optimum in the Long Run ***

See also: http://www.digitaleconomist.com/po_tutorial.html

The Digital Economist

Worksheet #6: Production and Costs

1. Given the following data, complete the table below:

$X = 18L^2 - L^3$ -- Production Function
$w = \$5.00$ -- Labor Costs/per unit
$FC = \$100.00$ -- Fixed Costs of Production

L	X	AP_L	MP_L	VC	FC	TC	ATC	MC	Stage of Production
0									
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									

a. How many units of labor would you hire if your goal is to minimize average total costs (ATC)?_________

b. Differentiate the production function to derive an equation for the marginal productivity of labor (MP_L):

c. Derive an expression for the average productivity of labor (AP_L):

d. Find the quantity of labor input where average productivity is a maximum (i.e., where $AP_L = MP_L$):

e. Find the quantity of labor input (to be hired) if your goal is to maximize profits given a market price (P_x) = \$0.08/unit. What is the dollar amount of profits in this case?

The Digital Economist, Worksheet #6

2. Given the following production function:

$$X = 30L^2 - 2L^3$$

a. Derive the Average Product and Marginal Product functions:

b. Given a wage rate of $48.00 per unit of labor, derive the average variable cost function and compute average variable costs for 8 units of labor. What is the corresponding level of output for this quantity of labor input?

c. Using a market price of $0.50/unit and the above wage rate of $48.00, derive and differentiate the profit function with respect to labor (assuming that labor is the only factor of production) and find the profit maximizing amount of labor input.

3. Given the following Cobb-Douglas production function:

$$X = \mathbf{10}L^{0.80}K^{0.20}$$

a. Does this production technology exhibit *Increasing/ Constant/ or Decreasing* returns to scale?__________ Explain:

b. Derive the average product function of labor:

c. Partially differentiate the above production function to find the marginal productivity of labor (MP_L) and Capital (MP_K). Holding one input constant, does this production function exhibit *diminishing/constant/or increasing* marginal productivity ________________

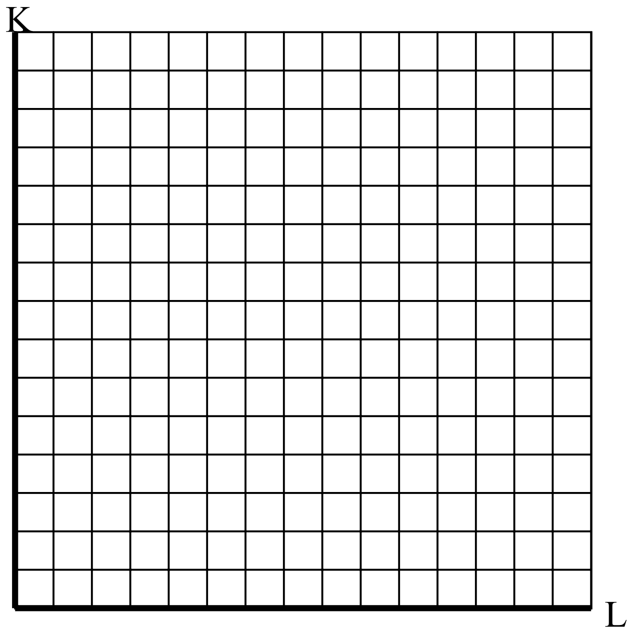
d. Calculate the output elasticity of labor input :

The Digital Economist, Worksheet #6

4. Given the following production function:

$$X = KL$$

a. Plot production isoquants for X = 24, X = 36, X = 48, & X = 72.



b. Does this production function exhibit: *Increasing/ Constant/ or Decreasing* returns to scale?_________________

c. If the cost function is defined by:

$$C = 3L + 12K \quad \text{(i.e., } w = \$3 \text{ and } r = \$12)$$

find the optimal amount of labor and capital for 36 units of output:

d. By how much will costs increase if output is doubled to 72 units?________ Do costs also double? ______________________ Explain: