The Digital Economist

Lecture 5 – Consumer Behavior

PREFERENCES

In taking a closer at market behavior, we need to examine the underlying motivations and constraints affecting the consumer (or households). We begin with the assumption that consumer behavior is guided by the desire to maximize the utility from the purchase, acquisition, and consumption of goods and services. The objective function of the consumer is written as:

$$\max \mathbf{U} = f(x,y)$$

The arguments on the right-hand side 'x' & 'y' represent measurable quantities of different goods or services. Unfortunately the term on the left-hand side of the expression, **utility** '**U**', is neither observable nor measurable. Thus we have to resort to the notion of individual *preferences* for goods and services to indirectly represent the utility (satisfaction) gained from consumption of these items.

We will make several assumptions about these preferences:

- individuals are able to make choices and rank their preferences for different goods and services
- individuals are rational in the choices they make.
- more of a particular good is preferred to less.
- additional units consumed provide less additional satisfaction relative to previous units consumed (*the more you have of a particular good, the less satisfaction you receive with additional consumption of that same good*).

The first assumption states that given several goods '**a**', '**b**', and '**c**', a consumer can define his preferences for these goods and put these preferences in some type of order. For example '**b**' may be *preferred* to '**a**', and '**a**' may be *preferred* to '**c**'. We summarize this assumption by saying that preferences are complete.

The second assumption states that if '**b**' is *preferred* to '**a**' and '**a**' is *preferred* to '**c**' then it must be true that '**b**' is *preferred* to '**c**'. This is known as the transitivity condition.

The third assumption is straight-forward in that greater quantities provide greater levels of satisfaction to the individual. This is known as non-satiation.

The last assumption states that consumers prefer bundles (or combinations) of goods and services that contain some variety of those goods rather than extreme bundles that contain large amounts of just one particular good. This is the concept of diminishing marginal utility.

If we consider two goods: *books* and *movies*, as shown in the left diagram of **figure 1** below. Both goods are desired by a given consumer (known as **economic goods** rather than **economic bads**). Points **a, b, c, d, e** each represent different combinations of these two goods.

From assumptions 1 and 2 we find that the consumer will decide on one of the following:

- **c** $\succ$ **b**, a preference for the bundle with more movies
- **b** $\succ$ **c**, a preference for the bundle with more books
- **c** ~ **b**, indifference '~' between a bundle that contains more movies and fewer books and the bundle with more books and fewer movies.

In the case where preferences for the two goods are defined, it must be the case that one good will provide more satisfaction (utility) relative to the other good. When indifference is the case, it must be true that the two bundles provide equal levels of satisfaction.

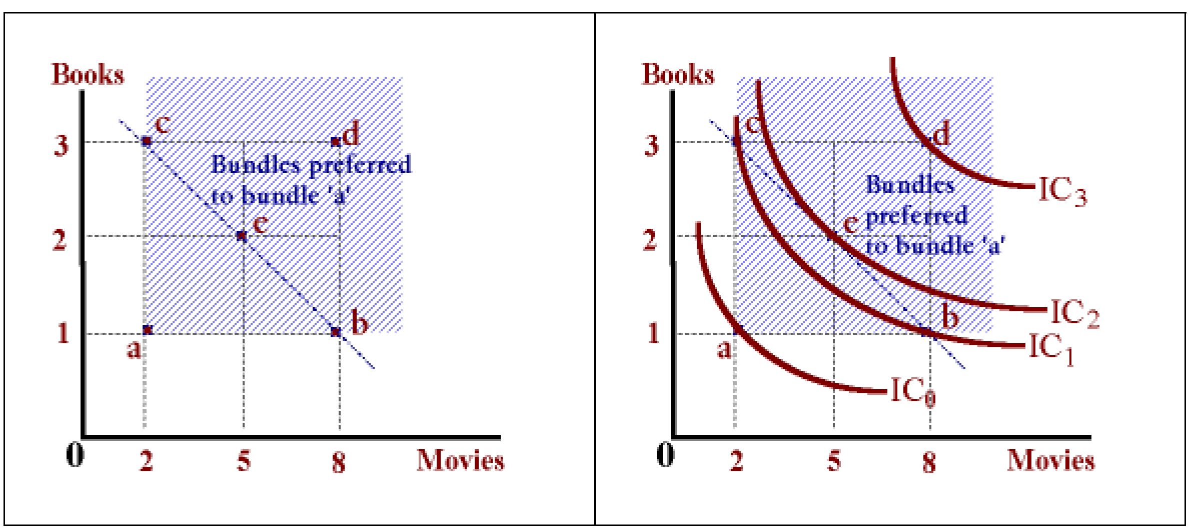
From our third assumption we can state that:

d $\succ$ **b** $\succ$ **a** and **d** $\succ$ **c** $\succ$ **a**

Finally the fourth assumption allows for comparison between the two extreme bundles '**b**' and '**c**' and an average bundle '**e**'. In this case if bundles **b** and **c** provide the same level of satisfaction then bundle **e** (which represents an arithmetic average of the former, i.e.,

e = θ **b** + (1 - θ)**c**   for (0 < θ < 1), will be preferred.

Figure 1, Preferences and Indifference Curves



Using these notions with respect to preferences, we can define a mapping that includes additional bundles of books and movies. This mapping is shown with the addition of the curves in the diagram on the right of figure 1. These curves, known as **indifference curves** represent combinations of the two goods that provide equal levels of satisfaction. All points on IC_1 represent bundles of books and movies that provide the same level of satisfaction as bundle **b** (8 books, 1 movie) or bundle **c** (2 books, 3 movies). All bundles on IC_2 provide more satisfaction than bundles included on IC_1 that provide more satisfaction than bundles on IC_0.

The position and general shape of these curves are defined through *assumptions* 1 and 2. In addition, assumption 2 prevents these curves from intersecting. For example, suppose that IC_1 and IC_2 intersected at point **b**. This would imply that:

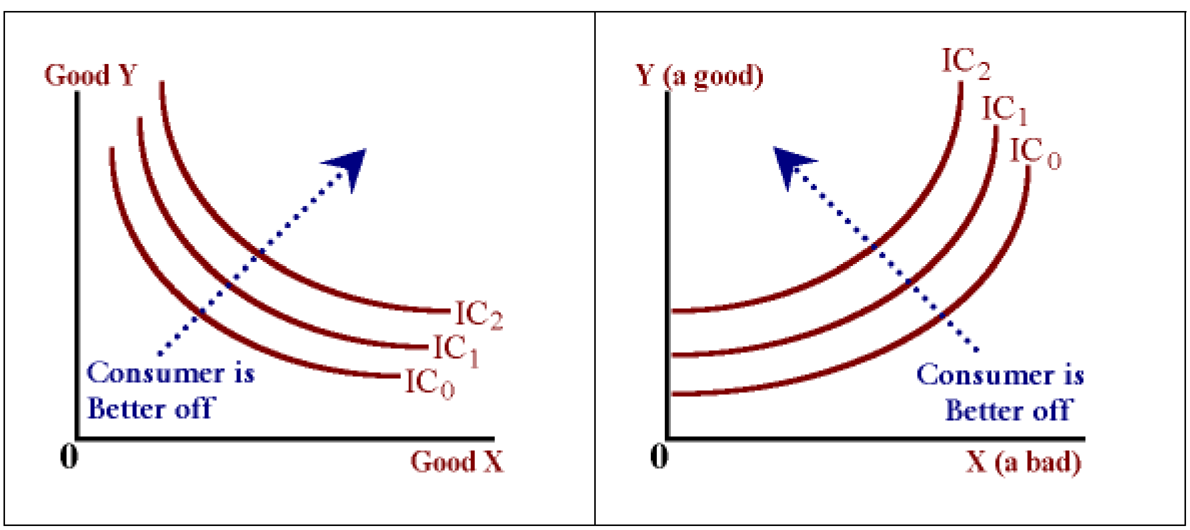
$$\mathbf{c} \sim \mathbf{b} \sim \mathbf{e}$$

but **e** is contains more books and movies than certain points on IC_1 (points to the interior of **e**) such that **e** must be preferred to these points as well as point **c**. Behavior in the case of intersecting indifference curves would be inconsistent and irrational.

These curves are downward sloping consistent with *assumption* 3 (if they were upward sloping, horizontal, or strictly vertical they would violate the condition of more is preferred to less). Finally *assumption* 4 (averages are preferred to extremes) leads to the convexity of the curves-- given $\mathbf{e} \succ \mathbf{c} \sim \mathbf{b}$ implies that IC_1 must contain points to the interior of **e**.

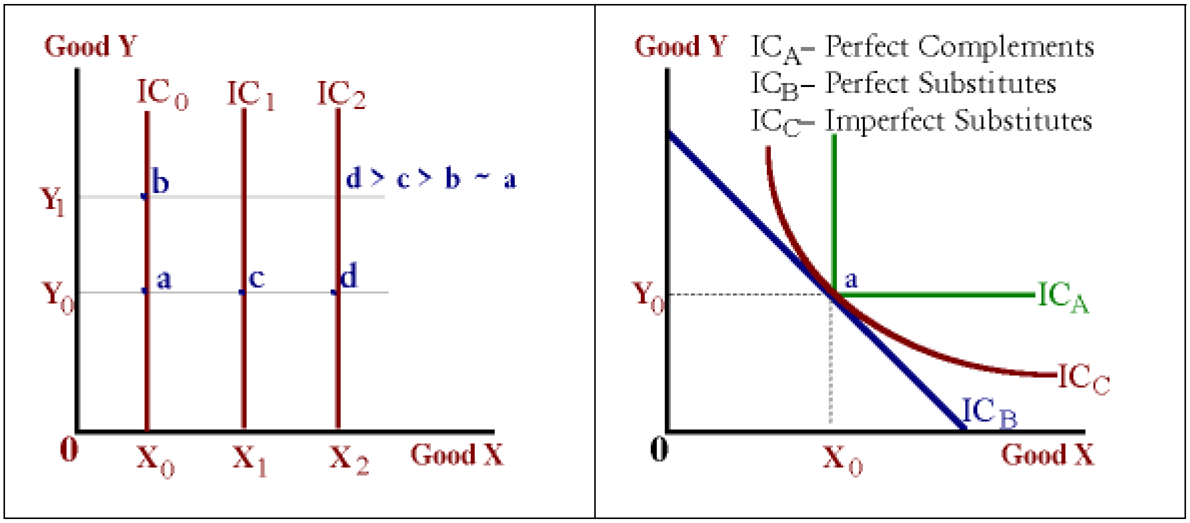
In different models these indifference curves can be used to identify preferences for combinations of: different products, consumption spending in the present and in the future, work-time and leisure time, or financial risk and return.

Figure 2, Different types of preferences



In the first two cases, we are talking about two goods where the indifference curves are downward sloping and movement to the north-east in the diagram indicates the individual is better off (figure 2, left). In the latter two cases we are talking about one good (*something preferred*) and one bad (*something to avoid*) where the indifference curves would be upward sloping and movement to the north-west or south-east (towards the good and away from the bad) indicates the individual is better off (figure 2, rght).

Figure 3, Extreme cases

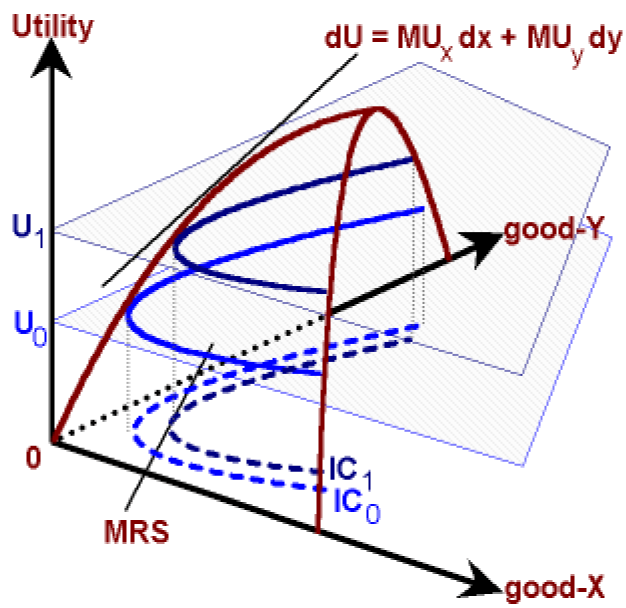


In figure 3, above, we find examples of extreme preferences for an individual consumer. In the left diagram, we have lexicographic preferences, where the consumer only gains satisfaction from one of the two goods. In this example, more of good-X makes this individual better off where as this person is indifferent between more or fewer units of good-y. In the right diagram we have a traditional indifference curve '**IC_C**' a two extreme cases with respect to the convexity of the curves. In the case of '**IC_A**' , the two goods are perfect complements for one-another (i.e., skis and ski boots). An increase in satisfaction only occurs if these two goods are acquired on proportional quantities. In the case of '**IC_B**', the two goods are perfect substitutes (i.e., nickels and dimes) such that consumer will only purchase one or the other good depending one the ratio of marginal utilities and relative prices.

UTILITY

Given a utility function: **U** = f(x,y), we can diagram this as a utility surface with the two goods measured along the base and utility measured as the height as shown in figure 4:

Figure 4, the Utility Surface (*diminishing marginal utility*)



This utility surface is a concave function based on our assumptions of diminishing marginal utility. Changes in elevation represent increasing levels of utility. Traversing the surface such that we hold elevation constant help to identify one particular indifference curve as a convex set in the x-y plane.

Using this utility function, we calculate separate expressions for the marginal utility of good-x and good-y:

$dU/dx = MU_x$ -- *holding 'y' constant*

or

$dU_x = MU_x dx$ -- *the equivalent of climbing the hill by traveling due east (along the 'x' axis).*

and

$dU/dy = MU_y$ -- *holding 'x' constant* or

$dU_y = MU_y dy$ -- *the equivalent of climbing the hill by traveling due north (along the 'y' axis).*

These **marginal utilities** represent a measure of the slope or steepness of the hill as we traverse in the due-east or due-north directions. With respect to the notion of utility, we can think of these marginal utilities as the rate by which consumption of either good is transmitted (or converted) into additional into utility or satisfaction.

Combining these separate transmission mechanisms, we can identify how utility changes with consumption of one or both goods:

$dU_{total} = dU_x + dU_y$

$= MU_x dx + MU_y dy$ -- *climbing the hill by moving in the north-east direction*.

Holding utility constant (**$dU_{total} = 0$**) or traversing the surface, as we would be doing along a given indifference curve, allows us to write:

$\mathbf{0} = MU_x dx + MU_y dy$

or

$dy/dx = -MU_x/MU_y$

Thus, the slope of an indifference curve at a given point in the x-y plane represents a measure of the ratio of marginal utilities which is also known as the **marginal rate of substitution** (**MRS**). This measure represents the rate at which one good must be substituted for another in order to keep the level of utility constant.

$\mathbf{MRS} = -MU_x/MU_y$

A CONSUMER OPTIMUM

A consumer optimum represents a solution to a problem facing all individuals -- *maximizing the satisfaction* (utility) from consuming different goods and services *subject to* the constraint of household income and product prices. This problem can be described as follows:

$max\ \mathbf{U} = f(x,y)$

s.t.

$P_x(x) + P_y(y) \leq I$

In this problem, the objective function is unobservable leading to the use of the assumptions about consumer preferences and diagrammed through the use of indifference curves. From our understanding of the utility function and utility surface we defined the slope of an indifference curve as:

the **Marginal Rate of Substitution** = $MRS^{xy} = -MU_x/MU_y$

With respect to the budget constraint, all variables and *are* observable and thus in defining an optimal solution for the consumer, we assume that this optimum lies somewhere on the constraint. This budget constraint can be written in several ways. First we can write it as a **budget set** '**B**':

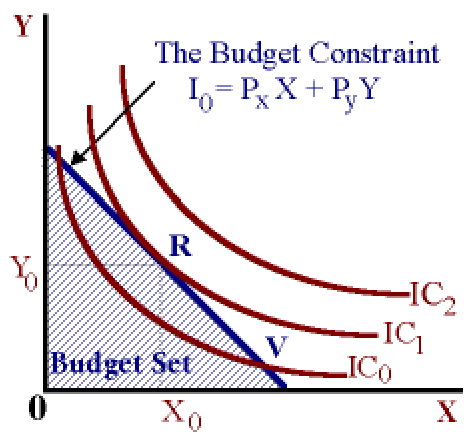
$\mathbf{B} = \{x,y \ \varepsilon \ \mathbf{R}^2 \mid x,y \geq 0; P_x(x) + P_y(y) \leq I_0\}$

This budget set represents all combinations of the two goods that are attainable to the consumer given his level of income and the market-determined prices of these goods. Second, we can write it as a budget constraint expressed as an *exact equality* in intercept-slope form:

$y = I_0/P_y - (P_x/P_y)x$

The slope of this budget constraint is a **relative price** (the price of good-x *relative* to the price of good-y) where a change in any price, either in absolute or relative terms, will lead to a rotation of this constraint. Both this budget constraint and budget set are shown in figure 5.

Figure 5, A Consumer Optimum



In this diagram, we can note that many different bundles of 'x' and 'y' on IC_0 are within this budget set and thus attainable. However, any point in the interior of the budget set represents an inefficient use of income. Point **V** on this same indifference curve does represent an *efficient* use of income however, the consumer can do better. At this point the slope of the budget constraint is greater than the slope of the indifference curve...

Point **V**: $P_x/P_y > MU_x/MU_y$ *or* $MU_x/P_x < MU_y/P_y$

At this point the marginal utility per dollar spent on good-x is less that the marginal utility per dollar spent on good-y. This consumer can increase his level of satisfaction by reallocating his income to buy more of good-y (*thus MU_y will decrease given our assumption of diminishing marginal utility*) and buy less of good-x (*MU_x increases*). This reallocation of income can be seen as a movement along the budget constraint from point **V** to point **R**. It is at point **R** that the consumer has found an optimum on **IC_1** where:

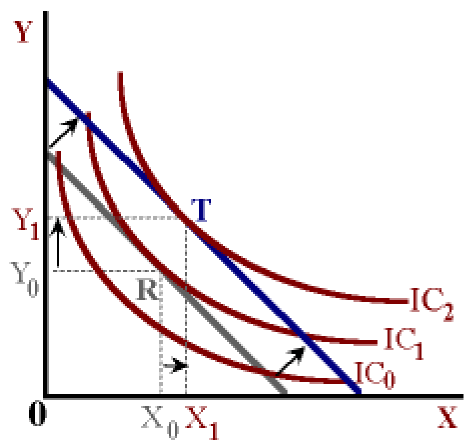
Point **R**: $MU_x/MU_y = P_x/Py$ *or* $MRS^{xy} = P_x/P_y$

This is our condition for a consumer optimum. Note that any bundle on **IC_2**, although providing a greater level of satisfaction, lies entirely beyond the budget set and thus could never include a solution to the problem facing the consumer.

Exogenous Shocks

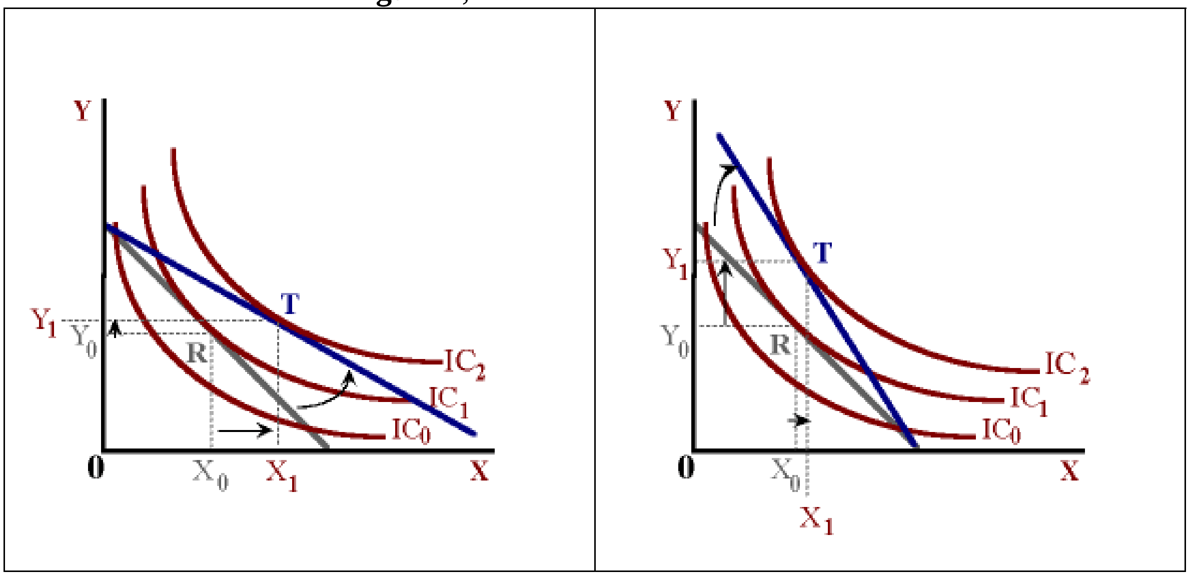
In figure 6, we can examine the effect of an increase in consumer income. This change can be seen in the right diagram above as a parallel outward shift in the budget constraint (*the slope remains the same since relative prices have not changed*). This increase in income increases the size of the budget set making a greater number of consumption bundles attainable to the consumer. This increase in the size of the budget set implies that the consumer will be *better-off* as defined with the new consumer optimum at point **T**.

Figure 6, Exogenous Shocks – An Increase in Consumer Income:



With this increase in income, the consumer chooses to consume *more of both goods* indicating that they are both **normal goods** to that individual. It is possible that with this type of shock, the consumer will choose to purchase *more* of one good and *less* of the other (a movement from **R** to **T** in the northwest or southeast direction). In this case one good is **normal** and the other an **inferior good**. It must be true that at least one good in the consumption bundle is a normal good. If all goods were inferior, then an increase in income would lead to a consumer optimum in the interior of the budget set.

Figure 7, Reductions in Market Prices



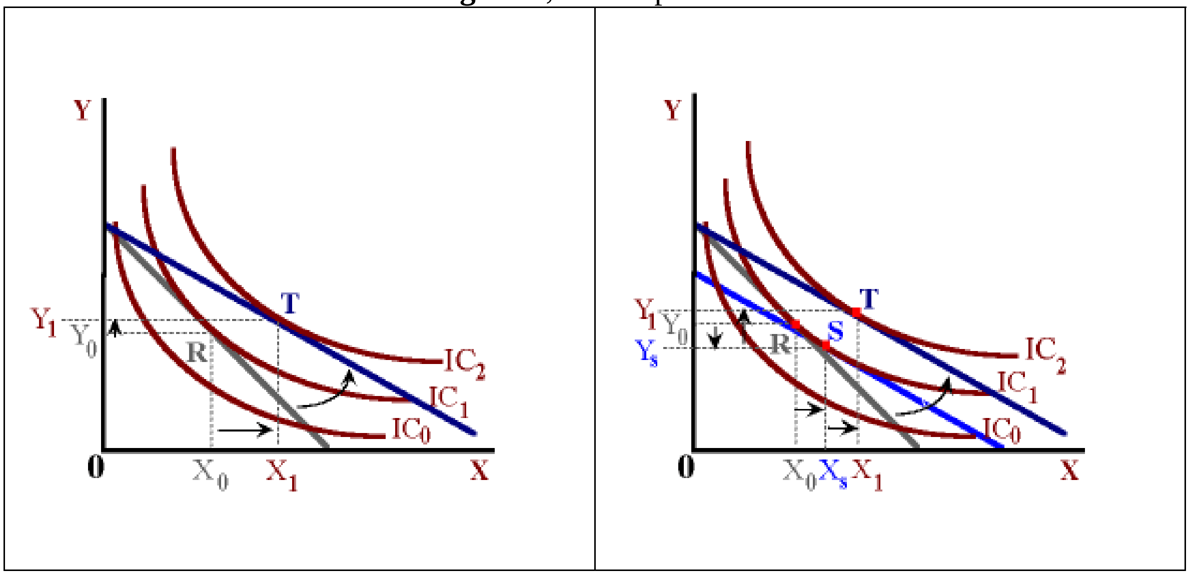
Additionally in figure 7, we see the effects of changes in market price. In the case of a *decrease in the price of good-x* the budget line rotates outward (*left diagram*) and the price ratio declines (*good-x is less expensive in absolute and relative terms, good-y is more expensive in relative terms*). This outward rotation also leads to an increase in the size of the budget set such that the consumer should be better off. We find that with this particular price change, the consumer is buying more of good-x and more of good-y as defined by a new consumer optimum at point **T**. A decrease in the price of good-y (*right diagram*) will also rotate the budget line, this time about the horizontal axis.

With the reduction in the price of good-x, the increase in the amount of good-x consumed is expected as it is now cheaper to the consumer. However the increase in consumption of good-y requires some explanation.

Price Decomposition

A change in the price of one particular good has two effects on consumer behavior. First, is the **substitution effect** where the consumer *substitutes towards that good* that is *relatively cheaper* (good-x) and *away from that good that is relatively more expensive* (good-y). Based on the substitution effect alone, we would expect the consumer to buy more of good-x and less of good-y. In the case of the latter good, this is not the case. A decrease in the price of any good in the consumption bundle also leads to *an increase in purchasing power*. The impact of this change is known as the *income effect* where, with this increase in purchasing power, the consumer will buy more **normal goods** and fewer **inferior goods**. In figure 8, we find that the consumer may be substituting away from good-y but he is also using the increase in purchasing power to actually buy more of that good.

Figure 8, Decomposition



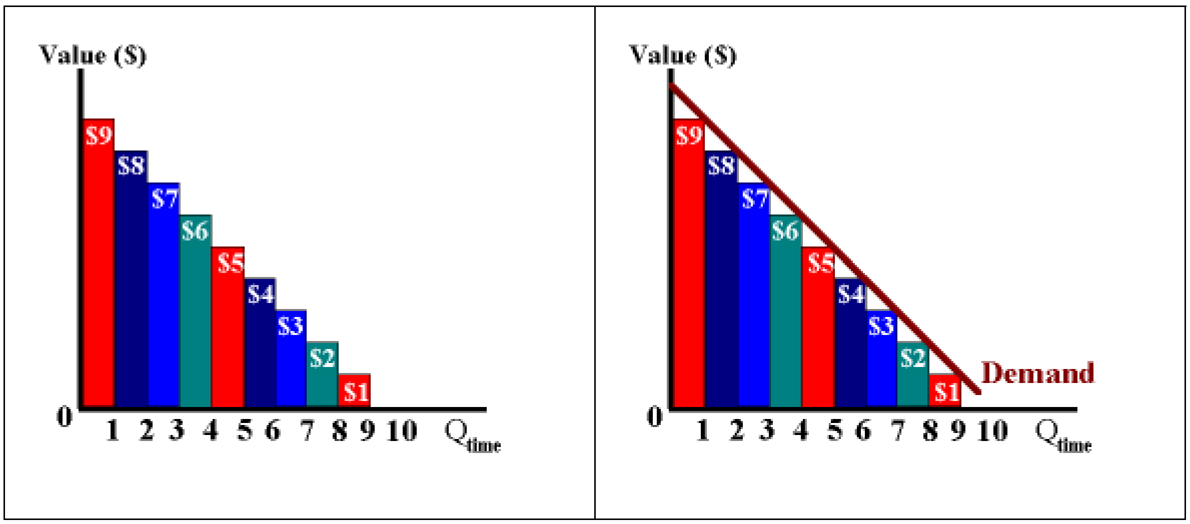
These two effects can be graphically decomposed from the price reduction. We define the **substitution effect** as a comparison of the old price ratio (P_x/P_y) and the new (Px*/Py) holding the level of utility constant. We can see this substitution effect in figure 6, by looking at the tangency of the original budget constraint (*the old price ratio*) at point **R** and a different tangency on the same indifference curve **IC_1** defined by the new price ratio at point **S**. The **income effect** is defined as a comparison between the two levels of satisfaction defined by **IC_1** and **IC_2** holding relative prices constant. Comparing the two levels of utility acts as a proxy for changes in purchasing power (similar to a change in the size of the budget set). This can be seen as the parallel shift from point **S** on **IC_1** to point **T** on **IC_2**. We find in this example that the consumer is substituting towards good-x and away from good-y and, in addition, he is using the increase in purchasing power to buy more of both goods.

CONSUMER SURPLUS

When analyzing changes to a consumer optimum given changes in the market price of a particular commodity, we often speak of the consumer being better or worse off. What is missing in this analysis is the ability to quantify changes in individual satisfaction due to these price changes. One method used to measure these welfare changes is through the use of a concept known as consumer surplus. This method compares the value of each unit of a commodity consumed against the price of that commodity. Stated differently, *consumer surplus measures the difference between what is person is willing to pay for a commodity and the amount he/she actually is required to pay*.

Consumer demand is a measure of willingness to pay. As shown in the diagram below, consumers often value each additional unit consumed less that previous units (i.e., *the concept of diminishing marginal utility*). For example, suppose that the good in question is monthly consumption of gasoline. According to the diagram below (left) , the consumer would be willing to pay $9 for the first gallon rather than to without. This first gallon would be used for essential driving activities. Each successive gallon has a value to the consumer of $1 less than previous units (2^{nd} gal = $8.00, $3r^{d}$ gal. = $7.00, *and so on*) as needs are met and the consumer engages in driving more for pleasure and sight-seeing. The value that the consumer places on each gallon (unit) consumed is summarized by the individual **Demand** curve as shown in the diagram on the right.

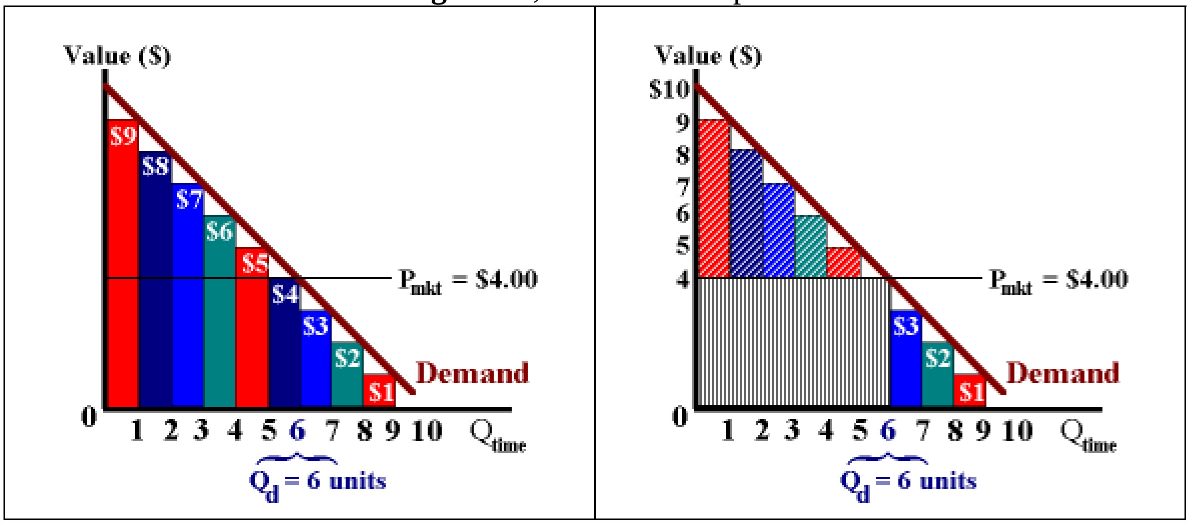
Figure 9, Diminishing Marginal Value



Consumers do have a choice in the purchase of gasoline or other goods. Either they could avoid market participation and spend nothing and receive nothing of value or they can purchase a certain quantity of this good and receive value over-and-above the market price. Consumer surplus represents the reward to consumers for participating in the market place.

If the market price of gasoline were $4.00 (for each and every gallon -- *the seller not being able to determine the value of each gallon sold*), the consumer would buy 6 gallons per month (see figure 10 below left).

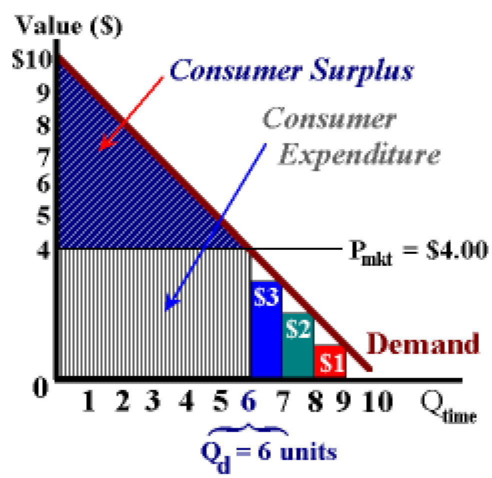
Figure 10, Consumer Surplus



In the diagrams above, we find that with a market price $\mathbf{P_{mkt}}$ of $4.00 per gallon and quantity demanded equal to 6 gallons, the **total value** of consumption is $39.00 ($9 + 8 + 7 + 6 + 5 + 4). Part of this value is given up in the form of **total expenditure** equal to $24.00 ($4 x 6gal) as shown by the **gray-shaded** area in the right diagram. The difference of $15.00 ($39.00-$24.00) represents consumer surplus as shown by the cross-hatched colored bars in the right diagram.

These measures of total expenditure and consumer surplus can neatly defined as geometric areas below a given demand curve. In the diagram below, the colored bars have been replaces with shaded areas allowing for divisibilities in consumption. Rather than restricting the consumer to purchase of 1, 2, 3, … gallons, we now allow that individual to be able to purchase fractions of a gallon.

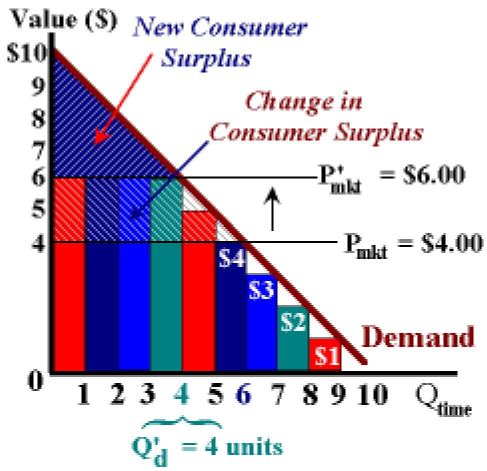
Figure 11, Consumer Surplus



We can use this measures to quantify the welfare effects of a change in market price by examining the corresponding changes in consumer surplus.

For example, suppose that in the above example, market price increases to $6.00 perhaps due to an increase in excise taxes. At this higher price, the consumer would be willing to purchase only 4 units of this product. In purchasing these 4 units, the consumer receives $30.00 worth of value ($9.00, $8.00, $7.00, $6.00) and spends $24.00 ($6.00 x 4 units). The difference of $6.00 is the new level of consumer surplus.

Figure 12, A Change in Market Price



By measuring the change in consumer surplus, we can begin to quantify the change in consumer welfare from the increase in gasoline prices:

CS_{before} = $15.00
CS_{after} = $6.00
ΔCS = -$9.00

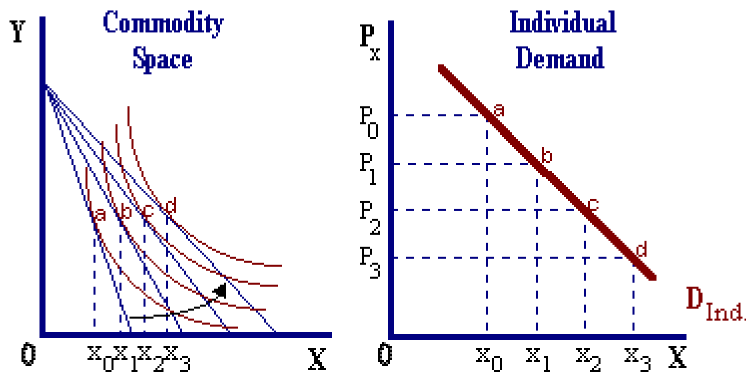
The $2.00 increase in the price of gasoline has led to a $9.00 reduction in consumer welfare.

Using the tools of indifference curve analysis, we can demonstrate that in increase in market price indeed makes the consumer worse off. By measuring the changes in consumer surplus, we can define *how much* worse off the consumer has become – a useful empirical tool for policy analysis.

INDIVIDUAL DEMAND CURVE DERIVATION

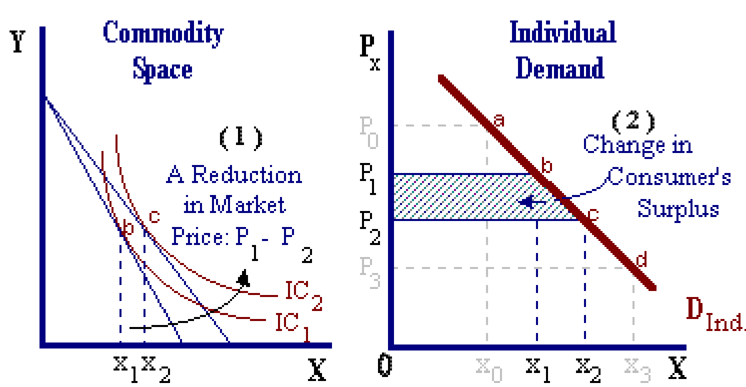
Individual demand curves can be thought of as a set of price-quantity combinations that each represent a separate consumer optimum for different market prices. This can be seen in the diagrams below:

Figure 13, Consumer Optimums and Individual Demand



Point 'a' in the left diagram represents a bundle of goods (x and y) that will maximize the consumer's level of satisfaction for a given set of market prices (P^0_x,P^0_y) and income (I_0). This same point in the right diagram represents an identical quantity of good-x demanded at a current price P0x. As the price of good-x declines the consumer is willing (**substitution effect**) and able (**income effect**) to purchase more of good-x. This inverse relationship between prices and quantity trace out the individual's demand for this commodity (points: ‘a’, ‘b’, ‘c’ & ‘d’). The slope of this individual demand relationship depends on the magnitude of the **total effect** of the price change and specifically the strength of the income effect. Stronger income effects (assuming normal goods) lead to flatter demand curves.

Figure 14, a Measured Change in Consumers Surplus



Additionally, this reduction in prices makes the consumer better off as shown in the tangency to higher indifference curves in the left diagram. This increase in consumer welfare can be measured by the corresponding change in consumer's surplus as shown in the above right diagram.

MARKET DEMAND

Finally, we can conclude our discussion by deriving a **market demand curve**. This market demand represents a (horizontal) summation of individual demand curves. Specifically, for each market price, individual consumers each have their own consumer optimums and corresponding demand for the good in question. We add up these demand for each possible market price to calculate the total quantity demanded in the market. For example:

	Quantities Demanded			
	Individual	**Individual**	**Individual**	**Market**
$\mathbf{P_{market}}$	**A**	**B**	**C**	**Demand**
$5.00	2	5	1	8
$4.50	4	6	4	14
$4.00	6	7	7	20
$3.50	8	8	10	26

Thus, we find that in the market, every time the price is reduced by $0.50, the total quantity demanded (market demand) increases by 6 units.

Be sure that you understand the following concepts and terms:

- Indifference Curve
- Marginal Rate of Substitution
- Marginal Utility
- Utility
- Completeness (of Preferences)
- Economic Bads
- Economic Goods
- Normal & Inferior Goods
- Non-Satiation
- Ordinal Utility
- Preferences
- Transitivity
- Utility Maximization
- Utility Surface
- Budget Constraint & Budget Set
- Relative Prices
- Consumer Optimum
- Income Effect
- Substitution Effect
- Total Effect
- Consumer Surplus
- Total Expenditure
- Total Value (of Consumption)
- Economic Welfare

Optimizing Conditions Discussed:

MRS = P_x / P_y (**MRS** *defined as* MU_x / MU_y)

or

$MU_x / MU_y = P_x / P_y$

or

$MU_x / P_x = MU_y / P_y$ $\Rightarrow$ ***A Consumer Optimum ***

See also: http://www.digitaleconomist.com/co_4010.html
http://www.digitaleconomist.com/cs_4010.html
http://www.digitaleconomist.com/d_d.html
http://www.digitaleconomist.com/co_tutorial.html

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Worksheet #4: Utility and Preferences

1a. Calculate the marginal utility for each quantity of the following two goods:

Bread (loaves)

Q_{week}	Utility	MU
0	0	
1	70	____
2	130	____
3	180	____
4	220	____
5	250	____
6	270	____
7	280	____
8	280	____

Tea (Cups)

Q_{week}	Utility	MU
0	0	
1	60	____
2	110	____
3	150	____
4	180	____
5	200	____
6	210	____
7	210	____

b. Does the Marginal Utility of these two goods *increase*, *decrease*, or *remain constant* with additional units of consumption?_________ Is this consistent with our assumptions about utility?_____________ Explain:

____

c. Assuming that you can trade one loaf of bread for one cup of tea ($P_{bread} = P_{tea}$), complete the following table assuming that you currently have 8 loaves of bread (your income) and no tea.

Bread	Tea	MU_{loss}	MU_{gain}	MU_{net}	Total Utility
8	0	-	-	_____	_______
7	1	_____	_____	_____	_______
6	2	_____	_____	_____	_______
5	3	_____	_____	_____	_______
4	4	_____	_____	_____	_______
3	5	_____	_____	_____	_______
2	6	_____	_____	_____	_______

d. At what combination of bread and tea will the consumer stop trading in order to maximize his/her utility from consumption of these two goods?

Bread:________ Tea:__________

Explain this result:___ ____

____

The Digital Economist, Worksheet #4

2. Given the following Utility function for two goods 'X' & 'Y':

$$\mathbf{U} = 3X^{0.5}2Y,$$

the marginal utilities of X & Y are:

$$MU_x = \frac{d\mathbf{U}}{dX} = 3Y/X^{0.50}$$

$$MU_y = \frac{d\mathbf{U}}{dX} = 6X^{0.5}$$

a. Derive an expression for the Marginal Rate of Substitution: MRS_{xy}

b. Given a consumption bundle of X & Y = (4,6), what is the value of the MRS_{xy}?

c. Which of the following price combinations correspond to a

consumer equilibrium given the results of part 'b'?_________

	P_x	P_y
A.	1	6
B.	3	4
C.	6	2
D.	1	3

d. If consumption of good 'X' falls to one unit (X = 2), how many units of good 'Y' are required to hold the level of utility constant?

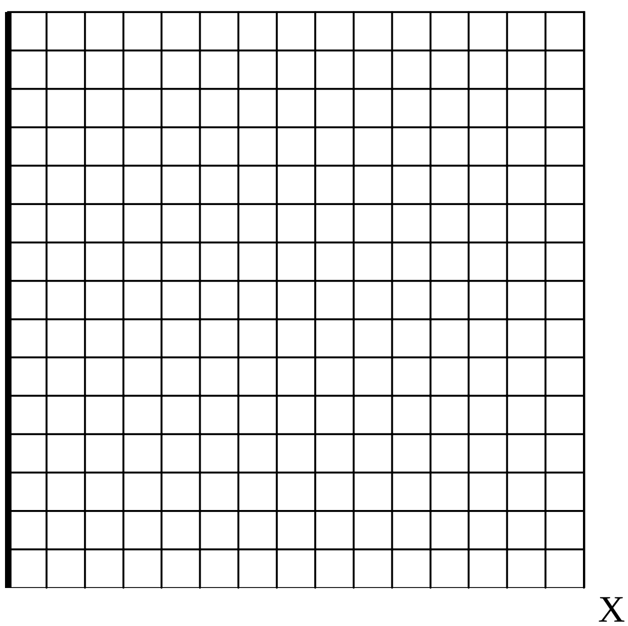
e. Given this change in the consumption bundle for goods 'X' & 'Y', has the MRS_{xy} increased or decreased?________________

The Digital Economist, Worksheet #4

3. A consumer has preferences represented by the following function:

$$\mathbf{U} = XY$$

a. Graph these preferences below for a constant level of utility of 24 utils:



b. Given the following income and prices:

$I = \$24$
$P_x = \$3$
$P_y = \$2$,

derive an equation for the budget constraint and plot this constraint on the diagram above.

c. Find the point of consumer optimum. What is the MRS at this point?

The Digital Economist, Worksheet #4

4. Given the following Utility function: $\mathbf{U} = X^{\alpha}Y^{\beta}$

and budget constraint: $I = P_xX + P_yY$

a. Calculate the corresponding Marginal Utilities for these two goods:

b. What restrictions can we make on the range of values for 'α' & 'β' given our assumptions of "*more is preferred to less*" and "*diminishing marginal utility*"?

c. Derive an expression for the Marginal Rate of Substitution:

d. Combine the MRS with the prices of these two goods to develop and equation for a(ny) consumer optimum:

e: Using a combination of the condition for a consumer optimum and the budget constraint, derive the demand equations for 'X' and 'Y' as a function of prices and income [i.e., $X = f(P_x,P_y,I)$ & $Y = f(P_x,P_y,I)$]:

f. Repeat steps **a-e** using the following utility functions:

$\mathbf{U} = (XY)^{0.50}$
$\mathbf{U} = \ln(XY)$
$\mathbf{U} = 4X + 3Y$