

The Digital Economist

Lecture 4 – Price Elasticity of Demand

An important characteristic of demand is the relationship among market price, quantity demanded and consumer expenditure. The nature of demand is such that a reduction in market price will usually lead to an increase in quantity demanded. Given that consumer expenditure is the product of these two variables, the effect of a price reduction will have an uncertain impact on this expenditure. In some cases a reduction in price will be more than offset by a large increase in quantity demanded -- a situation where demand is *price sensitive* or **price elastic**.

$$(P_{\text{mkt}} \downarrow) (Q_{\text{demanded}} \uparrow) = \text{Expenditure} \uparrow$$

In other cases, the reduction in price is proportionally smaller than the change in quantity -- a situation where demand is *price insensitive* or **price inelastic**.

$$(P_{\text{mkt}} \downarrow) (Q_{\text{demanded}} \uparrow) = \text{Expenditure} \downarrow$$

This relationship between price and quantity (for a linear demand function) can be demonstrated in the table below:

Table 1-- Price, Quantity, Expenditure

	Market Price	Quantity Demanded	Consumer Expenditure
	\$20	0	\$0
A	\$18	1	\$18
B	\$16	2	\$32
C	\$14	3	\$42
E	\$12	4	\$48
F	\$10	5	\$50
G	\$8	6	\$48
H	\$6	7	\$42
J	\$4	8	\$32
K	\$2	9	\$18
	\$0	10	\$0

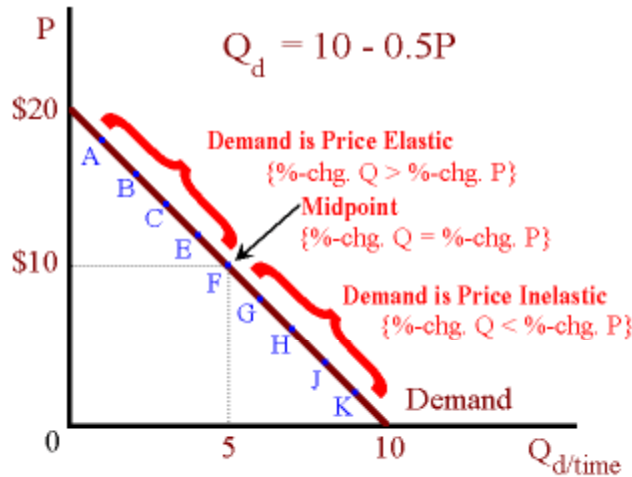
When the price falls from \$16 to \$14 -- a 12.5% reduction, quantity demanded increases by 50% (2 units to 3 units). Thus:

$$\% \Delta Q_d > \% \Delta P_{\text{mkt}}$$

and Expenditure *increases*. However, when the price falls from \$6 to \$4 (a 33.3% reduction -- same \$2 price change with a smaller base number), quantity demanded only increases by roughly 14% and expenditure *falls*.

On a linear demand function, all points on the upper half of the function represent price-quantity combinations where **demand is price elastic**. Points on the lower half represent combinations where **demand is price inelastic**.

Figure 1, Demand



Also note that at a price of zero (the horizontal intercept), the price elasticity of demand is equal to zero.

Numerically, the price elasticity of demand ' η_p ' represents the following ratio:

$$\eta_p = (\% \Delta Q) / (\% \Delta P)$$

such that:

if $(\% \Delta Q) > \% (\Delta P)$ then $|\eta_p| > 1.0$ and demand is **price elastic**

if the opposite is true then

$|\eta_p| < 1.0$ and demand is **price inelastic**

This relationship between price changes and expenditure can be summarized in the following table:

Elasticity:	Demand is Price Elastic: $ \eta_p > 1.0$	Demand is Price Inelastic: $ \eta_p < 1.0$
Price Reduction	Expenditure increases	Expenditure decreases
Price Increase	Expenditure decreases	Expenditure increases

On the demand-side of the market, elasticities can be calculated for any of the relevant exogenous variables. In all cases, elasticity measures the percentage change in quantity demanded relative to a percentage change in one of the exogenous variables.

Income Elasticity of Demand

Income elasticities measure the response in quantity demanded to a change in consumer income. In this case, the corresponding elasticity may be positive for normal goods, negative for inferior goods, or equal to zero for income neutral goods. This elasticity is computed as follows:

$$\begin{aligned}\eta_M &= \frac{(\% \Delta Q_d)}{(\% \Delta M)} \\ &= \frac{(\Delta Q/Q)}{(\Delta M/M)}\end{aligned}$$

If both numerator and denominator are of the same sign (both increase or both decrease) then the corresponding good or service is a **normal good**. If numerator and denominator are opposite in sign (one increases as the other decreases), then the good is an **inferior good**. Finally if the value of the numerator is zero (quantity demanded does not change with income), then the good is income neutral. The table below summarizes these results:

Income Elasticity	Type of Good
$\eta_M < 0$	<i>An Inferior Good</i>
$\eta_M = 0$	<i>An Income-Neutral Good</i>
$0 < \eta_M < 1.0$	<i>A Normal (necessity) Good</i>
$\eta_M > 1.0$	<i>A Normal (luxury) Good</i>

Cross-Price Elasticity of Demand

Cross-Price elasticity of demand measures the response of quantity demanded of one good to changes in the price of a second (related) good. This elasticity is computed as follows:

$$\eta_{xy} = \frac{(\% \Delta Q_d^x)}{(\% \Delta P_y)}$$

If the two goods are **substitutes** we would expect the following:

$$P_y \uparrow, Q_d^y \downarrow; Q_d^x \uparrow$$

In this case the price of good-y and the quantity demanded of good-x move in the same direction and thus the cross-price elasticity would be positive.

If the two goods are **complements**, then the relationship between the price of one good and quantity demanded of the other would be:

$$P_y \uparrow, Q_d^y \downarrow; Q_d^x \downarrow$$

Where in this case the price of good-y and quantity demanded of good-x move in opposite directions. The corresponding cross-price elasticity would be negative.

Finally if changes in the price of one good has no effect on the quantity demanded of the other, then the cross-price elasticity would be zero and the two goods are unrelated. The following table summarizes these results:

Cross-Price Elasticity	Goods 'x' & 'y' are:
$\eta_{xy} < 0$	<i>Complement Goods</i>
$\eta_{xy} = 0$	<i>Unrelated Goods</i>
$\eta_{xy} > 0$	<i>Substitute Goods</i>

Calculating Price Elasticities using a Specific Demand Equation

The formula for price-elasticity of demand may be rewritten as follows:

$$\begin{aligned} \eta_p &= \frac{(\% \Delta Q)}{(\% \Delta P)} \\ &= \frac{(\Delta Q/Q)}{(\Delta P/P)} \\ &= \frac{(\Delta Q) (P)}{(\Delta P) (Q)} \end{aligned}$$

In this last expression the $(\Delta Q/ \Delta P)$ term represents the *slope* of the demand equation. Thus if given the following linear demand equation:

$$Q_d = 10 - 0.50P$$

The slope in this case is a constant and equal to -0.50 . Combining this value with different price/quantity combinations (from the diagram above) we have:

Table 2—Elasticity Calculations

	Market Price	Quantity Demanded	Consumer Expenditure	Price Elasticity
B	\$16	2	\$32	$(-0.5)(16/2) = -4.0$
C	\$14	3	\$42	$(-0.5)(14/3) = -2.33$
E	\$12	4	\$48	$(-0.5)(12/4) = -1.50$
F	\$10	5	\$50	$(-0.5)(10/5) = -1.00$
G	\$8	6	\$48	$(-0.5)(8/6) = -0.67$
H	\$6	7	\$42	$(-0.5)(6/7) = -0.43$
J	\$4	8	\$32	$(-0.5)(4/8) = -0.25$

We can note that for price quantity combinations with prices above \$10, the (absolute value) of the elasticity calculation is greater than 1.0 – demand is price elastic. In order to interpret these values we can say, for example, that at point **B**; a 1% change in market price will lead to a 4% change in quantity demanded. At point **H**, we find that a 1% change in market price leads to a 0.43% change in quantity demanded – demand at this point is price inelastic.

Demand is not necessarily linear. The following demand equation represents a valid non-linear relationship between quantity demanded and market price.

$$Q_d^x = AP_x^\alpha$$

This expression is a valid demand relationship if the parameter α is strictly less than zero. In addition, the coefficient 'A', in this equation, is a measure all of the other exogenous influences on demand (*income, tastes and preferences, price of related goods, number of consumers in the market*). Changes in any of these exogenous variables will affect the value of this coefficient.

Thus, if we differentiate the non-linear demand equation given above and substitute, we have the following:

$$\begin{aligned} dQ/dP &= \text{the slope} = \alpha AP^{\alpha-1} \\ \eta_p &= \frac{\alpha AP^{\alpha-1}(P)}{(Q)} \\ &= (\alpha AP^\alpha)/Q = \alpha \end{aligned}$$

The parameter (*exponent*) ' α ' is the price elasticity of demand.

A more specific non-linear demand equation (*one where many of the exogenous variables are explicitly defined*) may be defined as follows:

$$Q_d^x = AP_x^\alpha M^\beta P_y^\phi P_z^\gamma.$$

It can be shown (through partial differentiation) that the parameters $\alpha, \beta, \phi, \gamma$ all represents various types of elasticity measures. That is:

$$\begin{aligned} \alpha &= \eta_p \text{ -- price elasticity of demand,} \\ \beta &= \eta_M \text{ -- income elasticity of demand,} \\ \phi &= \eta_{xy}, \quad \text{and} \\ \gamma &= \eta_{xz} \text{ -- cross-price elasticities.} \end{aligned}$$

Thus if we have the following (estimated) equation:

$$Q_d^x = 150P_x^{-0.75} M^{0.50} P_y P_z^{-1.25} .$$

we can state that demand for this particular good-x is **price inelastic** ($|\eta_p| < 1.0$), a **normal good** ($0.0 < \eta_M < 1.0$), a **substitute** with *good-y* ($\eta_{xy} > 0$), and a **complement** with *good-z* ($\eta_{xz} < 0$).

Be sure that you understand the following concepts and definitions:

(Consumer) Expenditure

- Linear Demand
- Price Elastic Demand
- Price Inelastic Demand
- Price Elasticity
- Complements / Substitutes
- Normal / Inferior Goods
- (Sales) Revenue / Consumer Expenditure
- Unitary Elastic Demand

See also: <http://www.digitaleconomist.com/elasticity.html>
http://www.digitaleconomist.com/elasticity_tutorial.html

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Worksheet #3: Price Elasticity

1. Given the following equations representing the market for wheat:

$$\text{Demand: } Q_d = 100 - 4P$$

$$\text{Supply: } Q_s = -20 + 2P$$

$$\text{Equilibrium: } Q_d = Q_s$$

- a. Calculate the *equilibrium price, quantity and expenditure / revenue* in equilibrium.
- b. In equilibrium is the demand for wheat price-elastic / price-inelastic or unitary-elastic? Show your calculations.
- c. Given your response to part 'b' if there is an increase in productivity in wheat production, will farm income (revenue) increase or decrease? Explain.
- d. Confirm your response to part 'c' by recalculating parts 'a' and 'b' using the following supply equation:

$$\text{Supply': } Q_s' = -30 + 2P$$

2. Given the following demand equation estimated by Pan Pacific Airlines for economy-class tickets:

$$Q_d = 150P_x^{-1.25} M^{1.50} P_y^{-0.50} P_z$$

where:

- Q_d -- Quantity Demanded of Economy-class tickets/week
- P_x -- Price of an Economy-class ticket
- M -- Per-capita Income
- P_y -- Price of a Bus ticket
- P_z -- Nightly Hotel Room Rate

a. Given this equation, Economy-class tickets are Price (*Elastic/ Inelastic*): _____, a (*Normal/Inferior*) _____ good, a (*Complement/Substitute*) _____ for Bus Tickets, and a (*Complement/Substitute*) _____ for Hotel Rooms.

b. Holding all other variables constant, if market price increases by 5%, Quantity Demanded will change by _____% and Total Consumer Expenditure will (*rise/fall*): _____.

c. What Exogenous variable(s) are indirectly measured by the constant term (150) in the above equation? _____

d. Suppose that Pan-Pacific Airlines is at full capacity (i.e., the airline cannot accommodate any additional passengers). However, GDP (national income) is expected to increase by 6% next year and population will grow by 2%. What will be the corresponding change in per-capita income? _____ Given this change in Per-Capital Income, by how much will Quantity Demanded change? _____

f. How exactly can management of Pan-Pacific Airlines offset this expected growth in demand? _____