The Digital Economist

Lecture 2 -- Production and Production Possibilities

An ideal place to start our discussion about economics is with the concept of production and production technology. Before there are any goods available for trade among individuals or for individual consumption, these good must be produced using available resources and technology.

THE PRODUCTION FUNCTION

Production refers to the conversion of inputs, the **factors of production**, into desired output. This relationship is about making efficient use of the available technology and is often written as follows:

$$X_i = f(L,K,M,R)$$

where X_i is the quantity produced of a particular (i^{th}) good or service and:

- **L** represents the quantity of labor input available to the production process.
- **K** represents capital input, machinery, transportation equipment, and other types of intermediate goods.
- **M** represents land, natural resources and raw material inputs for production, and
- **R** represents entrepreneurship, organization and risk-taking.

A positive relationship exists among these inputs and the output such that greater availability of any of these factors will lead to a greater potential for producing output. In addition, all factors are assumed to be essential for production to take place. The functional relationship $f(.)$ represents a certain level of technology and know how, that presently exists, for conversion these inputs into output such that any technological improvements can also lead to the production of greater levels of output.

Production in the Short Run

In order to better understand the technological nature of production, we distinguish between **short run** production relationships: where only one factor input may vary (typically labor) in quantity holding the other factors of production constant (i.e., capital and/or materials) and the **long run**: where all factors of production may vary. The short run allows for the development of a simple two variable model to understand the behavior between a single variable input and the corresponding level of output. Thus we can write:

$$X_i = f(L;\mathbf{K,M,R})$$

or

$$X_i = f(L)$$

For example we could develop a short run model for agricultural production where the output is measured as kilograms of grain and labor is the variable input. The fixed factors of production include the following:

1 plow
1 tractor.... capital
1 truck
1 acre of land
10 kilograms of seed grain

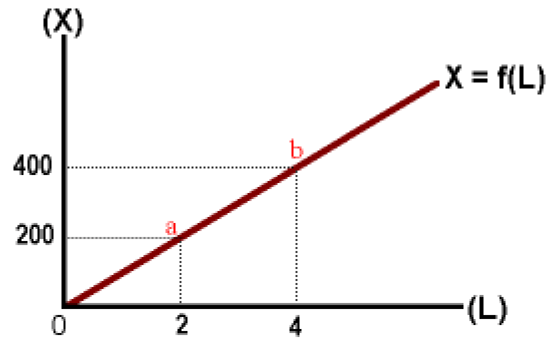
We might hypothesize the production relationship to be as follows:

Table 1, Production (*Constant Marginal Productivity*)

Input (L)	**Output (X_{grain})**	**MP_L**
0	0 kg	-
1	100	100
2	200	100
3	300	100
:	:	100
10	1000	100

In this example we find that each time we add one more unit of labor, output increases by 100 kg. The third column **MP_L** defines this relationship. This column measures the **marginal productivity of labor** -- a measure of *the contribution of each additional unit of labor input to the level of output*. In this case, we have a situation of **constant marginal productivity** that is unrealistic with production in the short run. Constant marginal productivity implies that as labor input increases, output always increases without bound -- a situation difficult to imagine with limited capital and one acre of land.

Figure 1, A Production Function (*Constant Marginal Productivity*)



A more realistic situation would be that of **diminishing marginal productivity** where increasing quantities of a single input lead to *less and less* additional output. This property is just an acknowledgment that it is impossible to produce an infinite level of output when some factors of production (machines or land) fixed in quantity. Numerically, we can model diminishing marginal productivity as follows:

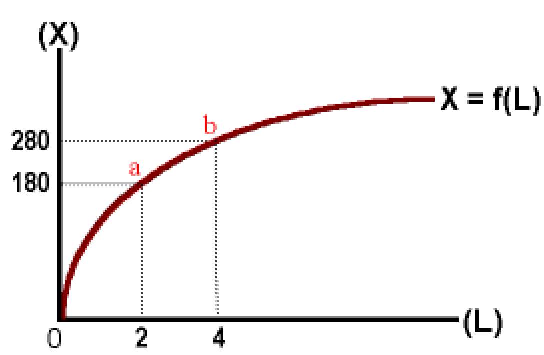
Table 2, Production (*Diminishing Marginal Productivity*)

Input (L)	**Output (X_{grain})**	**MP_L**
0	0 kg	-
1	100	100
2	180	80
3	240	60
4	280	40
5	300	20
6	300	0

In this case, additional labor input results in additional output. However, the contribution of each additional unit of labor is less than previous units such that the sixth unit of labor contributes nothing to output. With 5 or 6 workers, the available amount of land cannot support additional output.

A short run production relationship can be modeled in the diagram below. In this example, labor is the variable factor input and land, capital, and entrepreneurship are fixed in quantity. There is a positive relationship between labor input and output levels, however, as additional labor in used, less and less additional output is produced (click on the second button). The shape of this production function is consistent with the law of diminishing marginal productivity.

Figure 2, A Production Function (*Diminishing Marginal Productivity*)



PRODUCTION POSSIBILITIES

If we extend our model of production to two (or several) goods, we can develop a more realistic notion of production relationships. In a world of scarce resources, business firms producing different goods are competing for the same pool of factor inputs. In the short run labor is available for the production of one or a combination of goods. However, the desire to increase production of one good 'X' will come at the expense of another good 'Y' as labor or other resources are reallocated from the first good to the second.

In the table below, we can model this competition for resources between two goods: Apples and Bread. In this example, an additional units of labor directed to bread production allows for producing in the range of 0 – 210 units of bread. Separately, additional units of labor applied to apple production allows for the producing in the range of 0 - 185 units of apples.

Table 3, Production Possibilities: Bread and Apples
(7 units of Labor available)

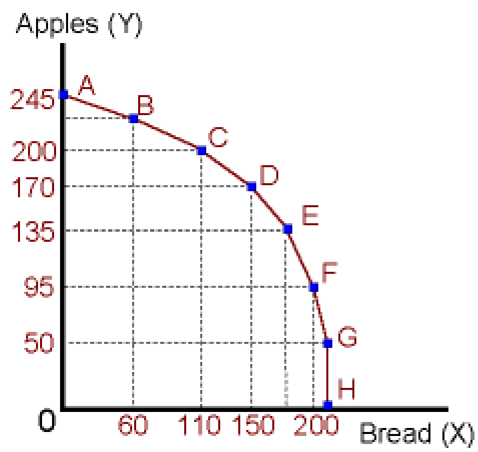
Pt.	X_{Bread}	$MP_{L,Bread}$	**Pt.**	Y_{Apples}	$MP_{L,Apples}$
A	0	-	**H**	0	-
B	60	60	**G**	50	50
C	110	50	**F**	95	45
D	150	40	**E**	135	40
E	180	30	**D**	170	35
F	200	20	**C**	200	30
G	210	10	**B**	225	25
H	210	0	**A**	245	20
	200	-10		260	15

Working with an assumption that the amount of labor input is fixed in supply at 7 units, this labor must be shared between bread and apple production. If we happen to be producing 245 units of apples (point **A**), then all 7 units of labor are being used for this purpose. The desire to produce bread requires a reallocation of labor from apple production to bread production. In the table above, we can show this as a movement from point **A** to point **B**. We are able to produce 60 units of bread (0-60 units) but at the sacrifice of producing 20 apples (245-225 units). These 20 apples represent the **opportunity cost** of bread production (i.e., *each unit of bread costs 20/60 unit or 1/3 apple*). If we move from point **B** to point **C**, the additional 50 units of bread (60-110), come at the "*cost*" of 25 apples (225-200 units) – *each bread costs 25/50 or 1/2 apple*. Because of diminishing marginal productivity in production of these two goods, the production of 50 additional units of bread requires that more and more apples are given up. Stated differently, we can say that the opportunity cost of producing bread is increasing. This is known as the ***Law of Increasing (Opportunity) Costs***.

The diagram below summarizes the numbers in the above table. Points on the maroon curve -- the **Production Possibilities Frontier** represent an efficient use of resources.

Points within the curve represent inefficient production levels -- resources and technology allow for producing more of good X, good Y, or more of both. Movements along the curve imply that a tradeoff exists in production when resources are scarce or fixed in supply. Finally points (combinations of the two goods) beyond the frontier are unattainable with existing levels of technology and resource availability.

Figure 3, The Production Possibilities Frontier



The Marginal Rate of Transformation
The diagram above helps us observe the slope of the PPF between any two points. This slope, is known as the Marginal Rate of Transformation (MRT), is a measure of the ratio of marginal productivity for each of the two goods.

Specifically:

$$MRT = MP_y / MP_x = \text{Marginal Cost}_x / \text{Marginal Cost}_y$$

given that: $\textit{Marginal Cost}_i = \textit{wage rate} / MP_i$

This ratio measures the opportunity cost of using resources in producing one good in terms of the alternative use of those resources used in the production of other goods. Given the role of diminishing marginal productivity; as resources are allocated away from good Y towards good X, the opportunity cost (|MRT|), of producing more of good X, increases.

If resources were to be allocated in the opposite direction, the same would be true -- the opportunity cost of producing more of good Y would also increase in terms of foregone production of good X.

See: The Digital Economist: http://www.digitaleconomist.com/ppf_4010.html#1

Opportunity Costs and Relative Prices

Suppose that we are producing at point **C** in the diagram below (left). If we transfer one unit of labor away from apple production to bread production, we must give up 30 units of apples and gain 40 units of bread. Thus the opportunity cost of each unit of bread is 3/4 (0.75) of an apple. Bringing **relative prices** into the picture, we might find that the price of apples 'P_{apples}' is \$2.00 and the price of bread 'P_{bread} is \$3.00. Or,

$$P^R = [P_x / P_Y] = [P_{bread} / P_{apples}] = 1.50.$$

Stated differently, (as a **Terms of Trade**), we find that we are willing to trade 1 unit of bread for \$3.00 and with that \$3.00, we could then acquire 1.5 units of apples. Or, one unit of bread is *worth* 1.5 units of apples given these prices.

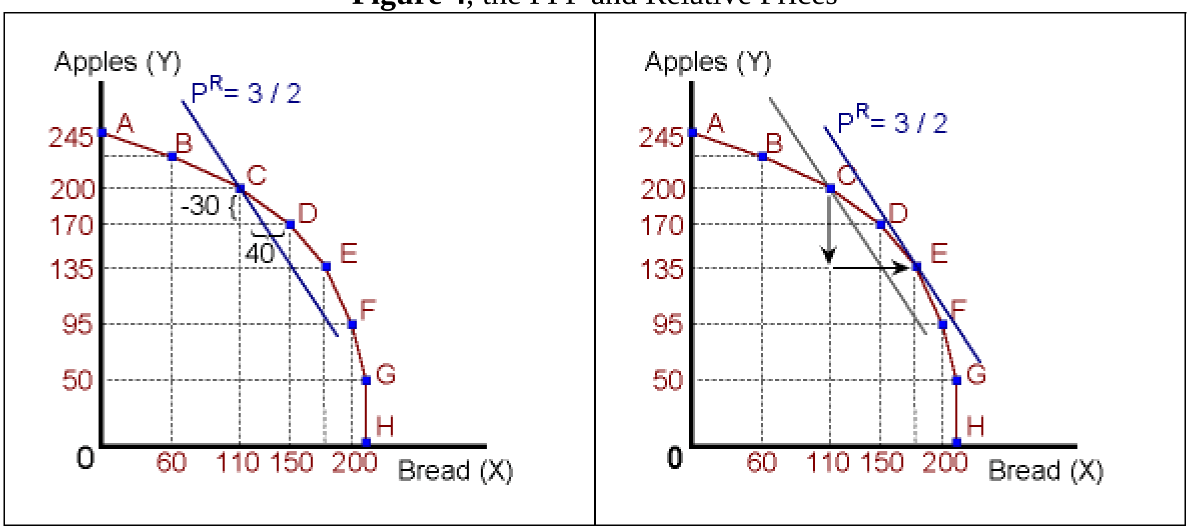
If we compare the relative price (value) of bread to the opportunity cost of bread, we find that the value of bread in terms of apples, at point **C**, is greater than the opportunity cost of producing bread:

One bread is *worth* 1.5 apples
One bread "*costs*" 0.75 apples.

or

$$P_{bread} / P_{apples} > MRT$$

Figure 4, the PPF and Relative Prices



From a social point of view, we should be allocating more resources towards bread production and away from apple production. With the reallocation of resources, the opportunity cost of bread production will rise. This reallocation should continue until the following is true:

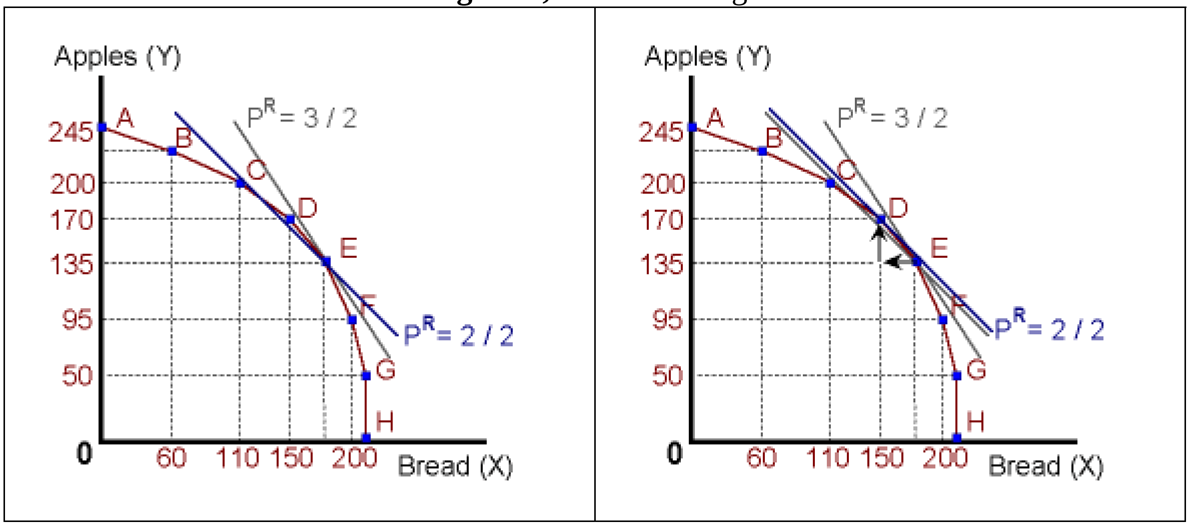
$$P^R = MRT$$

or

$$P_{bread} / P_{apples} = MC_{bread} / MC_{apples}$$

If the price of bread were to fall, say to $2.00 per unit, then the relative price of bread would be equal to 1.0 (i.e., each unit of bread is exactly equal to the value of one unit of apples) and $P^R \downarrow$. With this change, the value of bread, in terms of apples, is less than the opportunity costs of producing bread. In this case we should allocate resources away from bread production. This reallocation will cause the opportunity cost of bread to fall until it is just equal to the prevailing relative price.

Figure 5, A Price Change



Be sure that you understand the following concepts and terms:

- Production Function
- Production Technology
- Factors of Production
- Production in the Short run
- Production in the Long Run
- Fixed Factors of Production
- Variable Factors of Production
- Marginal Productivity (of Labor)
- Constant Marginal Productivity
- Diminishing Marginal Productivity
- Production Possibilities
- Production Possibilities Frontier (PPF)
- Relative Prices
- Opportunity Costs
- Marginal Rate of Transformation (MRT)
- Terms of Trade

Optimizing Conditions discussed:

MRT = P_x / P_y (**MRT** *defined as:* MC_x / MC_y)

or

$MC_x / MC_y = P_x / P_y$ … * **Economic Efficiency** *

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Worksheet #1: Production and Production Possibilities

1. Given the following production data, complete the following table:

Labor (L)	**Rice Kilograms**	**Marginal Product**	**Labor (L)**	**Houses Units**	**Marginal Product**
0	0	______	0	0	______
1	40	______	1	2	______
2	75	______	2	4	______
3	105	______	3	6	______
4	130	______	4	8	______
5	150	______	5	10	______
6	165	______	6	12	______
7	175	______	7	14	______
8	180	______	8	16	______
9	180	______	9	18	______
10	175	______	10	20	______

b. Graph these two production functions below:

Rice

0 **Labor**

Houses

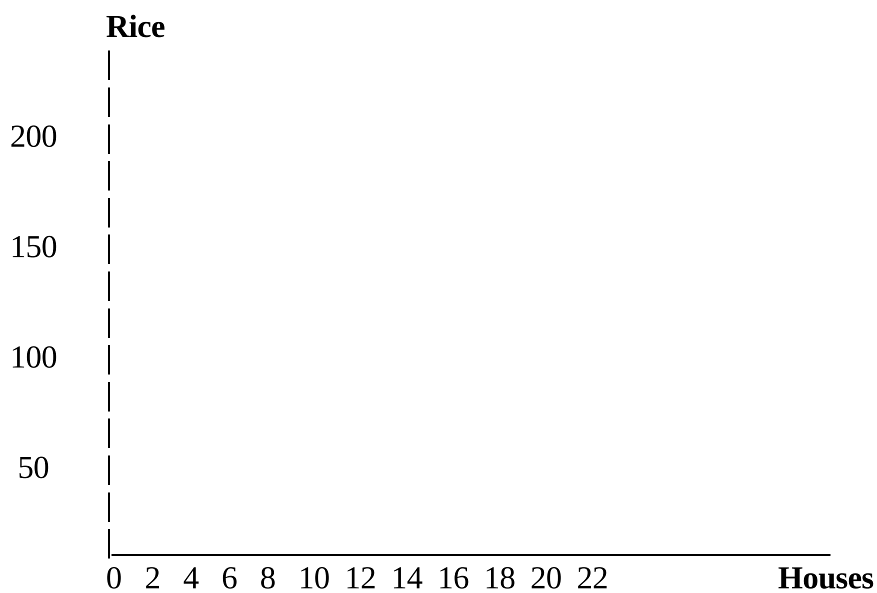
0 **Labor**

c. How would you characterize the production function for rice (diminishing, constant, or increasing marginal productivity)? ______________ How about the production function for houses?___________

d. Is it possible (in the short run) to produce 200 kg of rice?_______ Why or why not?

The Digital Economist, Worksheet #1

2. Given the production data on page 1, plot a production possibilities frontier for 6 units of labor to be allocated between rice and housing production.



b. What economic law determines leads to the curvature of the above production

possibilities curve? ____

c. Given that only 6 units of labor is available to be allocated between rice and housing production, is it possible to produce 100 kg. of rice <u>and</u> 5 houses?_______ Why or why not?____ Does this combination of rice and housing production represent an efficient use of resources?____________________

d. Assuming that we are currently producing 130 kg. of rice (using 4 units of labor) and 4 houses (using the remaining 2 units of labor). How many kg. of rice must be given up if we transfer one unit of labor away from rice production towards housing production?_______ How many additional houses could we build?____________ What is the *opportunity cost* of each additional house?_________ How are these *opportunity costs* measured?__________

e. Suppose that society values a kg of rice at $1,000 and values a house at $15,000. What is the relative price of a house as compared to rice (what is the *rice-value* of a house)? ______

f. Given the opportunity costs of an additional house (in part d) and the relative price of a house (in part e), should resources be allocated towards or away from rice production?_____

g. Show the impact of an increase in the size of the labor force from 6 to 10 workers on the above production possibilities curve.