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Lecture 6 – Investment Decisions

Investment is the act of acquiring income-producing assets, known as physical capital, either as additions to existing assets or to replace assets that have worn out (depreciated). These assets may be in the form of fixed nonresidential plant and equipment, housing (fixed residential) or business inventories. Decisions about the appropriate quantity of assets or capital are often based on profit-maximizing behavior of a private individual business firm producing goods and services or providing housing services.

The investment relationship may be written as follows:

$$\mathbf{J}_t = K^*_t - K_{t-1} + \delta K_{t-1}$$

where

- $\mathbf{J}_t$ = *Gross Investment (measured in units of new or replacement capital),*
- K^*_t = *Desired (profit-maximizing level) Capital Stock,*
- K_{t-1} = *Existing Capital Stock,*
- δ = *rate of depreciation*.

The difference in the first two terms ($K^*_t - K_{t-1}$) represents **net or new investment** and the last term (δK_{t-1}) represents **replacement investment**. **Gross investment** is just the sum of the two expressions.

By multiplying both sides of the equation by P_k (*the price of capital*) and factoring out 'K_{t-1}', we can write an expression for **Investment expenditure** ($\mathbf{I}_t$):

$$\mathbf{I}_t = P_k[K^*_t - (1-\delta)K_{t-1}]$$

where $\mathbf{I}_t = P_k\mathbf{J}_t$.

INVESTMENT DECISIONS and PROFIT MAXIMIZATION

The Net Present Value Approach

The key to understanding investment decisions is the determination of K*, the **desired capital stock**. This desired level is based, as stated above, on profit-maximizing behavior of the firm. There are several methods by which we can approach the profit-maximizing level of capital stock. First, we can analyze the **net present value** of different amounts of capital to determine which quantity gives the greatest discounted stream of profits. For example, given the following:

- P_k = $1000/unit,
- δ = 10%/year which implies that a unit of capital has a life of 10 years,
  We assume that the capital has no salvage value,
- r = 5%/annually -- this is the real market rate of interest,
- P_x = $5 -- the market price of a unit of output.

With this data we will use the following annuity factor computation:

Note: The Present Value of an Income producing asset that generates an annual income stream (revenue) of '**R**' is defined by the following formula:

$$\mathbf{PV}_{asset} = \Sigma_{[t = 1, N]} \mathbf{R}_t(1+r)^{-t}$$

or using the formula for a sum of a geometric series:

$$= (\mathbf{R}/r)[1 - (1+r)^{-N}]$$

as N → ∞, this expression reduces to R/r.

$$PV_N = (\text{Revenue})[1 - (1+r)^{-N}] \div r$$

$$= (\text{Revenue})[1 - (1.05)^{-10}] \div 0.05$$

$$= (\text{Revenue})7.722$$

Table 1, Net Present Value

Input K	**Output (units 'X")**	**Revenue [P_x(X)]**	**Costs [P_k(K)]**	**Present Value (N=10)**	**Net Present Value**
1	50	$250	$1930.50	$1000	$930.50
2	90	$450	$3474.90	$2000	$1474.90
3	**120**	**$600**	**$4632.00**	**$3000**	**$1632.00**
4	140	$700	$5405.40	$4000	$1405.40
5	150	$750	$5791.50	$5000	$791.50

In the above table we have a short run production relationship with capital being the only variable input. The input-output relationship is derived under assumption of **diminishing marginal productivity**. We find that three units of capital would generate the greatest net present value and thus the greatest profits.

The Marginal Revenue Product Approach

A second approach is the marginal approach where we compare the contribution to revenue by using one more unit of capital with the costs of acquiring that unit of capital. The contribution to revenue is known as the **Marginal Revenue Product** of capital and is calculated by multiplying the marginal product with the market price of the output produced:

$$MRP_k = MP_K P_x$$

The contribution to costs is known as the **Rental Cost of Capital** that includes borrowing costs (*or opportunity costs of using internal funds for investment expenditure*) and depreciation costs:

$$RCC = P_K(r) + P_K(\delta) = P_K(r + \delta)$$

The desired level of capital stock **K*** is that quantity where:

$$MRP_K = RCC$$

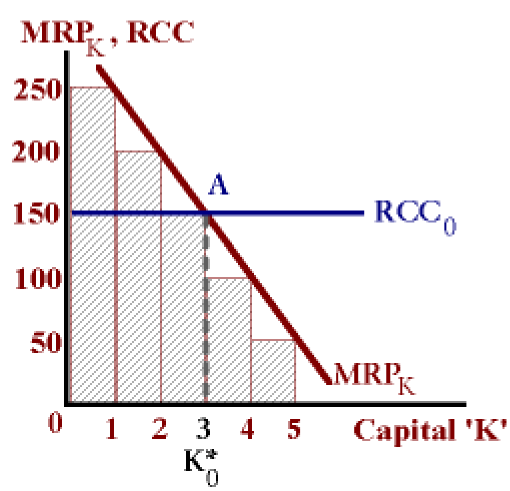
We will use similar data and a table for an example:

Table 2, MRP, RCC

K	Output	MP_k	MRP_k		RCC ($r = 5\%$, $\delta = 10\%$)
1	50 units	50	$250		$150
2	90	40	$200		$150
3	**120**	**30**	**$150**	**=**	**$150**
4	140	20	$100		$150
5	150	10	$50		$150

If the **MRP** exceeds **RCC** then profits will increase by acquiring additional units of capital (*contribution to revenue exceed contribution to costs*). If the opposite is true then the additional costs associated with one more unit of capital exceed the revenue generated and profits will decline. These relationships are shown in the diagram below:

Figure 1, MRP, RCC



See: http://www.digitaleconomist.com/capital.html *to experiment with changes in relevant parameters of this model.*

Similar to the **Net Present Value** approach, we find that with three units of capital the contribution to revenue of this third unit is just equal to the costs of acquiring and using that third unit--profits will be a maximum at this level of input.

Present Value of a Perpetuity Approach
A third approach is using a calculation similar to the present value of a perpetuity . We begin by using the marginal conditions above:

$$\mathbf{MRP = RCC}$$

$$\mathbf{MP_K P_x} = \mathbf{P_K(r + \delta)}$$

And we rearrange the terms:

$$[MRP_K - P_K(\delta)] \div P_K = \text{'}\Psi\text{'}$$

This term 'Ψ' represents the yield on the last unit of capital employed in the production process:

Table 3, Present Value of a Perpetuity

K	**Output**	**$MRP_K - P_K(\delta)$**	**P_K**	**Yield**	**r_{market}**
1	50 units	$250-$100	$1000	15%	5%
2	90	$200-$100	$1000	10%	5%
3	**120**	**$150-$100**	**$1000**	**5%**	**5%**
4	140	$100-$100	$1000	0%	5%
5	150	$50-$100	$1000	NA	5%

In this case, we find that the yield on the first two units of capital is greater than the market interest rate 'r_{market}' and thus may be acquired to earn profits over-and-above the borrowing costs. It is the third unit of capital where the yield is just equal to the market rate of interest -- no additional profits may be earned by hiring additional capital.

In all three approaches we find that the desired capital stock **'K*'** is equal to three units based on the productivity of capital, the rate of depreciation, the price of capital and the output being produced and, of greatest importance, market interest rates.

We can use a Cobb-Douglas form of the production function to solve for K* and combine this with the expression for investment defined above:

$$X = \mathbf{A}L^{\alpha}K^{\beta} \quad \textit{and} \quad MP_K = \beta X / K$$

given the profit-maximizing condition:

$$\mathbf{MRP_K} = \mathbf{RCC},$$

$$\mathbf{\beta(X/K)P_x} = \mathbf{P_K(r + \delta)}$$

or

$$\mathbf{K^*} = [\beta(P_x)X] \div [P_K(r + \delta)]$$

Inserting this expression for K* into the investment expenditure equation gives us:

$$\mathbf{I}_t = P_K\{ [\beta(P_x)X] \div [P_K(r + \delta)] - (1-\delta)K_{t-1}\}$$

or

$$\mathbf{I}_t = [\beta(P_x)X] \div (r + \delta) - (1-\delta)P_K K_{t-1}$$

where we find that Investment expenditure is <u>inversely related</u> to market interest rates '**r**' and the price of a unit of capital 'P_K', <u>positively related</u> to output prices 'P_x' and the productivity of capital (*as measured by the parameter 'β'*), and <u>undetermined</u> with respect to the rate of depreciation 'δ'.

TOBIN's - q

A different approach to understanding investment decisions is in using an expression known as **Tobin's-q**. This **q**-value represents a ratio between the market value of existing capital and its replacement cost:

$$\mathbf{q} = \text{Market Value} \div \text{Replacement Cost}$$
$$= \{(\text{Revenue})[1 - (1+r)^{-N}] \div r\} \div [P_K\, K]$$

If this value is greater than one, then the value of capital is greater than replacement costs and it would make sense to add or invest in more capital. It the **q**-value is less than one then the market value is less than replacement costs and business firms will allow existing capital to depreciate resulting in negative levels of investment. Given the inverse relationship between asset prices and market interest rates we can note that when these interest rates rise the market value of installed capital falls relative to replacement costs – dis-investment will occur.

The ACCELERATOR

A different approach to investment relative to the profit-maximizing model is that of the **accelerator mode**l. This model begins with the notion that a certain amount of capital is necessary to support a given level of economic activity. We can define this relationship as being proportional to RGDP '**Y**':

$$\Delta\mathbf{K}_t = \theta\Delta\mathbf{Y}_t$$
$$K_t - K_{t-1} = \theta(Y_t - Y_{t-1})$$

where 'θ' is known as the accelerator and represents a constant of proportionality between the two variables. If we restrict the definition of capital to only include business inventory stocks then we might note that:

$$0 < \theta < 1$$

Investment represents changes in these stocks such that:

$$\begin{aligned} \mathbf{I}_t &= P_k\{K_t - K_{t-1} + \delta K_{t-1}\} \\ &= P_k\{\theta\Delta \mathbf{Y}t + \delta K_{t-1}\} \end{aligned}$$

In order to understand investment behavior in this model, we need to look at the determinants of income or equilibrium level of expenditure:

$$\mathbf{Y}_t = C_t + I_t + G_t + NX_t$$

Defining consumption expenditure as being proportional to disposable income:

$Ct = b\mathbf{Y}_d = b(1 - t)\mathbf{Y}_t$ -- *where 'b' represents the marginal propensity to consume*.

We can then substitute:

$$\mathbf{Y}_t = b(1 - t)\mathbf{Y}_t + \theta P_k\{Y_t - Y_{t-1}\} + \delta P_k K_{t-1} + G_t + NX_t$$

For algebraic simplicity, we will eliminate the depreciation term and set 'P_k' equal to 1. Solving for $\mathbf{Y}_t$:

$\mathbf{Y}_t [1 - b(1 - t) - \theta] = A_t - \theta\mathbf{Y}_{t-1}$ -- where $A_t = G_t + NX_t$

Thus:

$$\mathbf{Y}_t^* = [1 - b(1 - t) - \theta]^{-1}(A_t - \theta\mathbf{Y}_{t-1})$$

The term in the brackets, $[1 - b(1 - t) - \theta]^{-1}$, is known as the **multiplier** which represents how a change in autonomous expenditure 'A_t' affects the equilibrium level of income. We will replace this term with a simple variable 'α' such that:

$$Y_t^* = \alpha(A_t - \theta Y_{t-1})$$

Using the following values for the parameters,

- the marginal propensity to consume -- $b = 0.75$,
- income tax rates -- $t = 0.20$,
- the accelerator -- $\theta = 0.10$, and
- $\mathbf{Y}_t = \mathbf{Y}_{t-1} = Y_{t-2} = 2500$.

we can compute a value for the multiplier to be equal to 3.333 thus:

$$\mathbf{Y}_t = 3.333[A_t - 0.10\mathbf{Y}_{t-1}],$$

and conduct the following simulation:

time:	**0**	**1**	**2**	**3**	**4**	**5**	**6**
$\mathbf{Y}_t$:	2500	2500	2833	2720	2760	2744	2750
A_t:	1000	1000	1100	1100	1100	1100	1100
$I_t = \theta\Delta\mathbf{Y}_t$:	0	0	33.33	-11.30	4.00	-1.60	0.60

We find, in this example, in time periods 0 and 1 the economy is in equilibrium with an income level of $2500. In time period 2, there is an autonomous shock (*an increase in government spending or perhaps an increase in net-export spending*) of $100. Working through the multiplier, the $100 change in autonomous spending leads to a $333 increase in income (*an example of the multiplier process*). Changes in income mandate increases in inventory to support this additional economic activity. However, after this initial shock, the economy oscillates towards a new equilibrium of $2750 by time period 6. Through time, with this cyclical behavior in income, there is also cyclical behavior in investment with additions taking being offset by reductions every other year. Eventually as the economy approaches the new level, the need for new investment approaches zero.

See: http://www.digitaleconomist.com/acc_modl.html *to experiment with the parameters of the accelerator model.*

The nature of this cyclical behavior is sensitive to the values of the marginal propensity to consume, tax rates (*which impacts the multiplier process*), and the value of the accelerator (*which affects both the multiplier and the rate of acceleration*). For example suppose we increase the value of the accelerator to 20%. The new value of the multiplier will be equal to 5.0:

time	**0**	**1**	**2**	**3**	**4**	**5**
$\mathbf{Y}_t$:	2500	2500	3000	2500	3000	2500
A_t :	1000	1000	1100	1100	1100	1100
$I_t = \theta\Delta Y_t$:	0	0	+100	-100	+100	-100

In this case the autonomous shock leads to an economy that oscillates forever between $2500 and $3000 with investment being +$100 every other year. It is the larger value of the multiplier relative to the size of the MPC that leads to this result.

This relationship between the accelerator and the MPC (or marginal propensity to spend when taxes are considered) helps explain the cyclical behavior of economic activity. We can summarize this relationship below:

Table 4, Oscillations in Real GDP

	Large θ	**Moderate θ**	**Small θ**
High propensity to Spend (b > 0.60)	*Explosive*	*Cyclical*	*Damped*
Moderate propensity to Spend (b<0.60)	*Explosive*	*Explosive*	*Cyclical/Damped*

Be sure that you understand the following concepts and terms:

- Marginal Productivity
- Marginal Revenue Product
- Rental Cost of Capital
- Real Rate of Interest
- Rate of Depreciation
- Gross Investment
- Net Investment
- Replacement Investment
- Desired Capital Stock
- Present Value
- Present Value of a Perpetuity
- Tobin's **q**
- the Accelerator
- the Multiplier

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Worksheet #4: Capital Accumulation

1. Given the following data:

Capital [**K**]	Output [**X**]	MP_K	Output Price (P_x)	MRP_K (MP_K x P_x)	Output Price (P_x')	MRP_K'
50	1000	70	$2.00	$140	$4.00	________
60	1600	60	2.00	120	4.00	________
70	2100	50	2.00	100	4.00	________
80	2500	_______	2.00	________	4.00	________
90	2800	_______	2.00	________	4.00	________
100	3000	_______	2.00	________	4.00	________
110	3100	_______	2.00	________	4.00	________
120	3150	_______	2.00	________	4.00	________

- The Price of a unit of capital (P_k) is $320
- The [expected] life of a unit of capital is 10 years
- The Optimal level of the capital stock '**K***' is defined where:

$$MRP_K = P_K(r + \delta)$$

- 'δ' represents the annual rate of depreciation

Complete steps a – d for a market output price 'P_x' of $2.00

a. If the real rate of interest 'r' is 15% (0.15), what is the optimal level of capital stock '**K***'?

b. If the real rate if interest falls to 2.5%, what is the new optimal level of capital?

- Investment is defined as: $I_t = K^*_t - K_{t-1} + \delta K_{t-1}$

c. If K_{t-1} is equal to 70 units of capital, what will be the level of *investment expenditure* for r = 15% at the existing rate of depreciation?

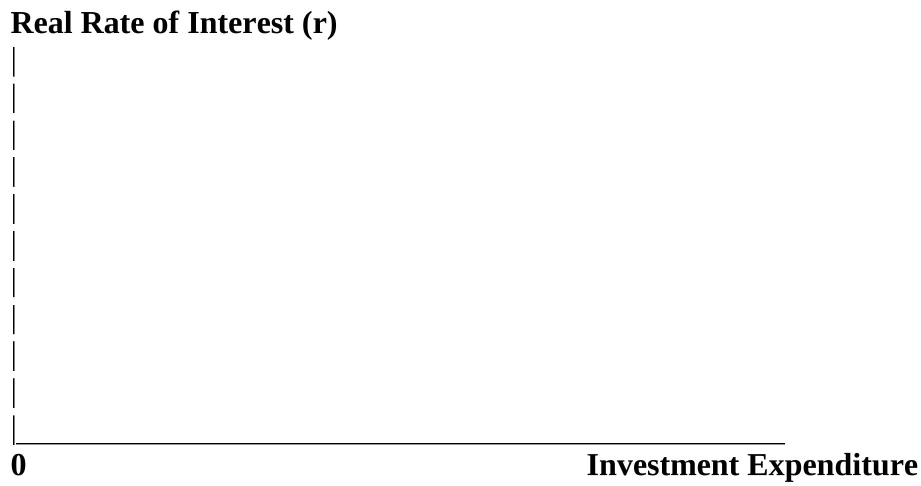
d. What will be the level of *investment expenditure* for K_{t-1} = 70, r = 15% and a rate of depreciation 'δ' of 0.225?

e. Repeat steps 'a-c' for an output price (P_x') of $4.00.

Worksheet #4, page 2

(Question #1, cont.)

f. Graphically show the relationship between investment expenditure and the real rate of interest. Specifically, show the level of expenditure for $K_{t-1} = 70$, $\delta = 0.10$, $P_x = \$2.00$, $r_0 = 15\%$, and $r_1 = 8.75\%$.



2. Suppose that production is defined by a Cobb-Douglas production function

$$Y = A\mathbf{L}^{1-\alpha}\,\mathbf{K}^{\alpha} \qquad \alpha = 0.30,\ \mathbf{A} = 100$$

a. Provide an economic interpretation for the parameters 'α' and '**A**'.

b. Does this production function exhibit *increasing returns*, *constant returns*, or *decreasing returns* to scale?____________________________ Explain.

Using the following expression for the *Rental Costs of Capital* (RCC), solve for the optimal level of capital stock '**K***' as a function of the real interest rate 'r':

$$RCC = P_k(r + \delta) \qquad = \$320(r + 0.10)$$

Remember that K is that level of capital where* $RCC = MRP_K$.

c. By how much does the optimal capital stock change when the interest increases from 'r = 10%' to 'r = 8%'?