INTEREST RATES – A BAROMETER of the REAL ECONOMY

Upon casual inspection of any business periodical, one will find that at any point in time there are many different nominal interest rates. The following table may be useful in organizing these different nominal interest rates that exist on any given day:

Table 1, Nominal Interest Rates at a point in time

Term	**No Risk U.S. Treasury**	**Low Risk (AAA-AA)**	**Medium Risk (A-BB)**	**High Risk (B-CC)**
Short-term (1-year or less)	1.80%	3.36%	3.95%	5.00%
Medium-term (1 - 10 years)	4.24%	4.46%	5.12%	8.00%
Long-term (10+ years)	5.54%	6.21%	6.89%	10.00%

Each column in the above table represents a different level of risk associated with a certain class of borrowers. This risk is also known as credit risk where different types of borrowers (or related projects) have different probabilities of being able to service their debt (make scheduled interest payments) and being able to repay the principal of the debt. These risk categories are commonly established by various credit agencies; the most popular being Standard & Poor's (listed above) and Moody's.

Each row represents different lending/ borrowing periods. **Short-term** lending corresponds to anytime between one day and one year. The **Medium-term** corresponds to a lending period between one and ten years. **Long-term** lending is with respect to debt contracts for a time period greater than ten years.

In the upper-left corner of the table is a short-term risk-free interest rate – the **T-Bill** (Treasury Bill) rate. This rate may be thought of as a core rate acting as a barometer of current macroeconomic conditions: the level of real economic activity and expectations about inflation. This nominal rate of interest (*looking forward*) on a short-term debt contract *(one year or less*) is developed as follows:

$$i_{T\text{-bill}} = r^* + E[\pi_t]$$

The first term '**r***' represents the **desired real rate of return** on debt and the second term '**E[π_t]**' represents the **expected rate of inflation** over the duration of the debt contract.

Over time, changes in the whole constellation of market interest rates may be attributed to changes either in the real desired rate 'r* or due to changes in inflationary expectations E[π_t]. Changes in this desired real rate reflects the behavior in the market for loanable funds. If the supply of these funds (*public and private savings*) exceeds the demand for these funds (*public and private borrowing*) then the desired rate should fall in reaction to a surplus of these funds. In periods of economic growth the opposite is true. The growing economy is sustained in part by increased borrowing activity for inventory investment and investment in new capital stock to allow for increased production to meet growth in aggregate demand. This desired real rate of return, therefore, represents the reward for lending to a productive and growing economy. Over time we would thus expect the following relationship to hold true:

$$r_t^* = E[\%\Delta RGDP]$$

This expression states that the real return to lending should be equal to the rate of real economic growth, that is, that the change in purchasing power made available in loan returns should be supported by growth in economic output.

Savings, Investment and the Real Rate of Interest

Savings represents the availability of funds to be lent via financial markets and behind those funds actual resources of the real economy. Domestic Investment Expenditure represents the borrowing and use of those funds. It is important to keep in mind that these funds being transferred from lender to borrower represent scarce resources. These resources are made available via the saver foregoing current consumption allowing these resources to be used for the creation, accumulation, and replacement of capital -- capital that will allow labor to be more productive in the future.

Measures of **National Savings** begins with the Income Identity:

$$\mathbf{Y} \equiv C + I + G + NX$$

Where '**Y**' represents both the (fixed) output of an economy and expenditure on that output at a point in time. Each component on the right-hand side represents the disposition of a nation's output. The first term '**C**' represents private consumption of goods and services similarly, the third term '**G**' represents the public consumption of goods and services. The '**I**' term represents private investment expenditure or the demand for resources in support of capital accumulation (plant and equipment, new residences and maintenance of inventories. Finally, the last term '**NX**' represents net export expenditure or the difference between exports (the foreign demand for domestic goods and services) and imports (*the domestic demand of foreign-produced goods and services*). For the United States, net exports are chronically in deficit such that borrowing is necessary from foreign businesses and institutions to support the excess of imports over exports.

Thus taking the above identity and rearranging terms:

$$\mathbf{Y} - C - G - NX = \mathbf{I_{[-]}(r)}.$$

The left-hand-side of this equation represent the **source of funds** (*savings*) and the right-hand-side, the use of those funds (*investment*). Additionally, we note that the demand for these funds are (negatively) related to the real rate of interest '**r**' – a relationship that will be explored in detail in a subsequent section below.

By subtracting and adding-in taxes '**T**' which represents the transfer of resources from the private sector to the public sector we have:

$$[\mathbf{Y} - T - C] + [T - G] + [-NX] = \mathbf{I(r)}$$

Each term in brackets now represents a separate type of savings defined as follows:

$\mathbf{S_{Private}} = [\mathbf{Y} - T - C]$ ... Private savings.

The term '**Y – T**' represents disposable income to the private sector available for expenditure '**C**' or the remainder Savings '$S_{private}$'.

$\mathbf{S_{Public}} = [T - G]$ ... Public Savings,

noting that if tax revenues '**T**' exceed government spending '**G**' then the public sector is running a **budget surplus** and the level of public savings is positive. If, however, government expenditure exceeds taxes collected, a **budget deficit** is the result such that the public sector must draw on the savings from other sectors of the economy.

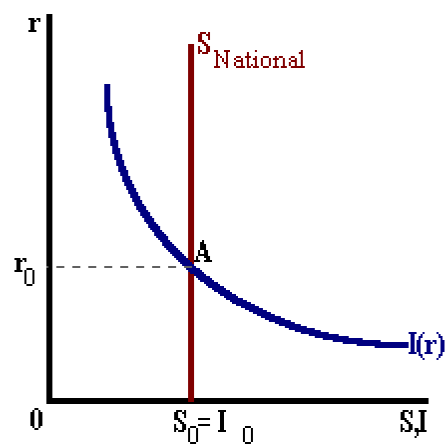
$\mathbf{S_{foreign}} = [-NX]$ ... Foreign Savings

Note that **NX** represents the **Current Account Balance** within the balance of payments. Thus, its negative, [**-NX**] represents the **Capital Account Balance** that, when positive in value, represents foreign lending to the domestic economy.

Adding these three terms together we have:

$$\mathbf{S_{National}} = \mathbf{S_{pvt}} + \mathbf{S_{pub}} + \mathbf{S_{foreign}}$$

Figure 1, Savings and Investment



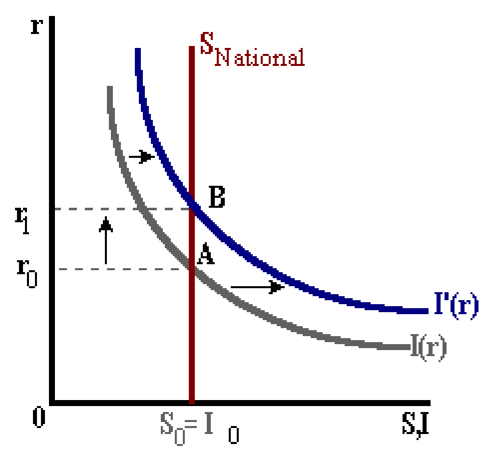
This level of savings is shown by the vertical line in the diagram to the right. The real rate of interest will adjust in competitive financial markets to bring National Savings into equality with Domestic Investment as shown by $\mathbf{r_o}$.

Given that this real rate of interest '**r**' is a reflection of the level of economic activity (the availability and demand of resources for capital accumulation), we can treat this value as a guide for the determination of the desired real rate of return **r***.

Note: the above model represents the **Primary Market** for loanable funds where actual borrowing and lending activity take place. In the section on **Financial Markets** we will distinguish between **Primary Market** activity and **Secondary Market** that allow for the trading of existing financial assets. Both markets are instrumental in the establishment of specific nominal interest rates.

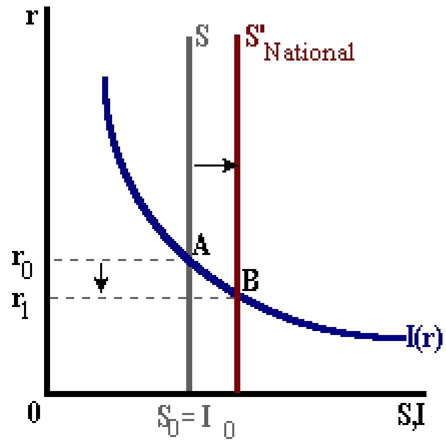
Various shocks can affect the **flow of funds** and thus the real rate of interest. For example, an increase in investment expenditure perhaps due to an <u>increase in the productivity of capital</u> or due to <u>growth in the real economy</u> will shift the Investment schedule to the right. This shock will lead to an excess demand for funds such that, for a given level of savings, real interest rates will increase. This increase in real rates will then lead to a movement along the new investment demand curve to point '**B**'. This higher real rate of interest is a reflection of greater competition for resources in support of capital accumulation.

Figure 2, An Investment Shock



Fiscal policy shocks can also affect the flow of funds and real interest rates. For example, suppose that Government Expenditure '**G**' decreases. Holding taxes constant, this will lead to an increase in **Public Savings** and **National Savings**. The savings function will shift to the right creating an excess supply of funds thus causing the real rate of interest to fall. This decline in the real rate will induce private businesses and individuals to increase their level of borrowing to finance new inventories, plant and equipment, and new residential structures.

Figure 3, A Fiscal Shock
A Reduction in Government Expenditure



Changes to the <u>tax rate</u> is a bit more complicated. Changes in taxes work through the Marginal Propensity to Consume to affect consumption expenditure and thus **Private Savings** indirectly as well as affecting **Public Savings** directly.

Note: The **Marginal Propensity to Consume 'MPC'** is a stable parameter that represents the fraction of each additional dollar of income devoted to consumption expenditure. For example if the MPC is equal to 0.80, then for each additional dollar of income received by an individual, his/her level of consumption expenditure increases by $0.80.

A Numerical Example

Suppose that we have the following equations:

C = 0.80(Y - T)	... consumption expenditure, MPC = 0.80
T = 0.20(Y)	... Tax Revenue, tax rate = 20%
G = 2,000	... Government Exp. (billions)
NX = -500	... Current Acct. Balance (deficit)
I = 2,600 - 100(r)	... Investment Exp.
Y = 10,000	... Real GDP (Output in billions) held constant.

Solving (given **Y** = 10,000), we find that:

T = 2,000
C = 0.80(10,000 - 2,000) = 6,400

and

S_{pvt} = Y - T - C = 1,600,
S_{pub} = T - G = 0 and,
$S_{foreign}$ = [-NX] = 500

thus,

$S_{National} = 1{,}600 + 0 + 500 = 2{,}100$

setting

$S_{National} = I(r)$

we have:

$2{,}100 = 2{,}600 - 100(r)$

and

$r_0 = 5\%$ and Investment expenditure = 2,100.

* * *

Now, if the tax rate were to be reduced to 10% ($t' = 0.10$):

$T' = 1{,}000$
$C' = 0.80(10{,}000 - 1{,}000) = 7{,}200$ and
$\Delta C = +800$

$S'_{pvt} = Y - T - C = 1{,}800$ and
$\Delta S_{pvt} = +200$

$S'_{pub} = T - G = -1{,}000$ and
$\Delta S_{pub} = -1{,}000$

$S_{foreign} = [-NX] = 500$ as before.

Thus,

$S'_{National} = 1{,}800 - 1{,}000 + 500 = 1{,}300$ and
$\Delta S_{National} = -800$

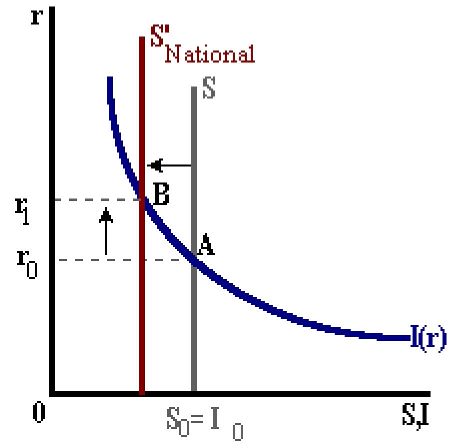
setting $S'_{National} = I(r)$ we have:

$1{,}300 = 2{,}600 - 100(r)$ or

$r_1 = 13\%$ and Investment expenditure = 1,300.

Reducing the tax rate from 20% to 10% has led to an increase in the real interest rate from 5% to 13%. With this interest rate increase, private Investment Expenditure will be reduced (from 2,100 to 1,300). A reduction in tax rates reduces **Public Savings** by an amount less than the offsetting increase in **Private Savings** – National Savings declines leading to an increase in the real rate of interest. This is shown in figure 4 below:

Figure 4, A Fiscal Shocks
A Tax Cut



Any event that leads to changes in savings or investment will affect the real interest rate and therefore the desired real rate of return. This change in ‘r*’ will be reflected in changes in nominal interest rates similar to those listed in **table 1**.

Inflationary Expectations

A second and frequent cause of changes to the structure of nominal interest rates would be due to changes in inflationary expectations.

An individual lending money in an inflationary environment will be repaid in dollars that possess less purchasing power upon maturity of the debt contract. An **inflation premium** is often built in to nominal interest rates protect against this loss of purchasing power. However, at the time the debt contract is developed the inflation premium is based on expected rates of future inflation. If these expectations differ from actual inflation rates during the life of the debt contract either the lender or borrower can be adversely affected.

The inflation premium is added to the desired real rate of interest 'r*' in the market determination of nominal market interest rates 'i_{market}' (*i.e., those interest rates published in the paper or posted on the wall at a bank*). This sum was reflected in the definition for the T-Bill rate above.

Changes in inflationary expectations tend to be a fairly complicated matter. One may hypothesize that current inflationary expectations are based on the history of past actual rates of inflation. A formal model that may help in understanding the development of these expectations is that of the **Adaptive Expectations** model. This model is based on the notion that economic agents develop forecasts of future inflation based on past actual rates adjusted for their own past expectations. Specifically, inflationary expectations are calculated by using a weighted average of past actual 'π_{t-1}' and past expected inflation '$E[\pi_{t-1}]$':

$$E[\pi_t] = \theta\ \pi_{t-1} + (1-\theta)E[\ \pi_{t-1}] \qquad \textit{where } 0 < \theta < 1$$

Past actual inflation 'π_{t-1}' represents a reflection of the economic environment – one where prices may be rising or falling. Past expected inflation '$E[\pi_{t-1}]$' serves as a proxy for the magnitude and trend in the history of inflation rates over the recent past.

By algebraically rearranging this equation we have:

$$E[\pi_t] = E[\pi_{t-1}] + \theta\{\ \pi_{t-1} - E[\pi_{t-1}]\}$$

where the term in the brackets represents the forecast error made by the economic agent in attempts to determine the previous rate of inflation. From this second equation current inflationary expectations are defined to be the sum of the rate previously expected and this forecast error. The rate by which economic agents adapt to accelerating inflation depends on the value of the weight 'θ' assigned to past expected inflation in developing current inflationary expectations.

This model is explains the possibility that economic agents may adapt slowly to a changing inflationary environment. This may have been the case in the late 1960's and early 1970's. During the 1960's, the inflation rate was relatively low in the 2-4% range. Basically, during this period time inflation was not considered to be a major economic problem. Thus in the next decade when actual inflation began to creep up towards the double-digits, many individuals and institutions were surprised. Forecasts of future inflation (*based on recent historical experience*) consistently lagged behind an accelerating actual rate of inflation.

> *See:* The Digital Economist: http://www.digitaleconomist.com/aex_4020.html
> to learn more about the workings of the Adaptive Expectations Model.

There are other sources of information available and used by economic agents in developing these inflationary expectations. One is based on the rate of money growth 'M' in excess of the long run rate of growth in output 'Y^R':

The Quantity Equation

A popular identity defined by Irving Fisher is the **quantity equation** commonly used to describe the relationship between the money stock and aggregate expenditure:

$$\mathbf{MV \equiv PY}$$

Note: An "identity" is an expression that is is true by definition such as the following:

a triangle = a three sided geometric figure

There is no debate about this validity of this equality, its truth comes from the nature of the definitions used.

The terms on the right-hand side represent the price level (**P**) and Real GDP (**Y**). Taken together these two terms represent Nominal GDP or a measure of the total spending that takes place in an economy in a given time period.

On the left-hand side, **M** represents some measure of the money supply, perhaps $\mathbf{M_1}$, and '$\mathbf{V_1}$' represents the velocity of this monetary measure. Velocity represents the number of times money changes hands in support of the total spending in an aggregate economy. Thus, we might more accurately state the equation as follows:

$$\mathbf{M_1V_1 = PY^R},$$

denoting the use of $\mathbf{M_1}$, its corresponding velocity and Real GPD '$\mathbf{Y^R}$'. For example, if in 2001, Nominal GPD ($\mathbf{PY^R}$) was equal to roughly \$10 trillion. In that same year, $\mathbf{M_1}$ was measured at roughly \$2.2 trillion with a corresponding velocity of 4.5. or

$$\$2.2(4.5) = \$10.0$$

The \$2.2 trillion of money was used in support of \$10.0 trillion of expenditure.

If we chose to use $\mathbf{M_2}$ as our monetary measure then the expression would be:

$$\mathbf{M_2V_2 = PY^R}$$

The truth of the expression does not change. Even though, we find ourselves using a broader definition of money, and corresponding velocity measure will be smaller.

The quantity equation can be transformed as follows:

$$(\mathbf{M} + \Delta M)(\mathbf{V} + \Delta V) = (\mathbf{P} + \Delta P)(\mathbf{Y} + \Delta Y)$$

or

$$\mathbf{MV} + \mathbf{V}\Delta M + \mathbf{M}\Delta V + \Delta V\Delta M = \mathbf{PY} + \mathbf{Y}\Delta P + \mathbf{P}\Delta Y + \Delta P\Delta Y$$

by definition **MV** = **PY** so these terms cancel. In addition, if we assume that the changes in the variables are relatively small, the product of changes will be close to zero such that we can write:

$$\mathbf{V}\Delta M + \mathbf{M}\Delta V = \mathbf{Y}\Delta P + \mathbf{P}\Delta Y$$

dividing the left-hand side by MV and the right-hand side by PY (again the two expressions are equal to each-other by definition), we have:

$$\frac{\cancel{V}\Delta M}{M\cancel{V}} + \frac{\cancel{M}\Delta V}{\cancel{M}V} = \frac{\cancel{Y}\Delta P}{P\cancel{Y}} + \frac{\cancel{P}\Delta Y}{\cancel{P}Y}$$

or

$$\mathbf{\%\Delta M + \%\Delta V = \%\Delta P + \%\Delta Y^R}$$

In this last expression, the first two terms represent growth in the money stock and growth in velocity respectively. On the other side of the expression we have the sum of the rate of inflation, and the rate of Real economic growth. If we are able to assume that velocity is a numerical constant (*its value determined by institutions and habits that see little change over time*), this expression can be written as follows:

$$\mathbf{\%\Delta P = \%\Delta M - \%\Delta Y^R}$$

The implications of this expression are that if growth rates in the money stock exceed the rate of real economic growth, inflation will be the result. The money stock growing by a smaller amount as compared to the rate of economic growth will lead to deflationary pressures in the aggregate economy. Price stability implies that growth in the money stock should match the [expected] rate of growth in a particular economy. Inflationary expectations are thus affected by differences that may exist between these two growth rates.

$$\mathbf{E[\pi_t] = \%\Delta M - \%\Delta Y^R}$$

The Phillips Curve

Another source of information is based on what is known as the **Phillips Curve** relationship – a relationship between unemployment rates and inflationary pressure in the economy:

$$\mathbf{\pi_t = -\beta(u - u^*)}$$

In the above expression 'u' represents the actual rate of unemployment and 'u*' represents the **natural rate of unemployment**. This latter value represents *that rate of unemployment where there is no upward nor downward pressure on the price level*. If the actual rate of unemployment exceeds the natural rate then there is slack in labor markets with little upward pressure on wages, costs and thus prices. If, however, the actual rate is below the natural rate then labor markets are relatively tight and a tendency exists for wages to be pushed upward.

Thus any event that causes an economic agent to revise his/her inflationary expectations will be translated into corresponding changes in nominal interest rates.

The Ex-Post Real Interest Rate

Because of the difficulty in the development of inflationary expectations, it is possible for the actual rate of inflation to differ significantly from these expectations. When these forecast errors do occur, the actual real rate of interest may differ from the desired rate.

At the termination of the debt contract an ex-post **real rate of interest** 'r' can be developed as follows (an expression known as the Fisher Equation):

$$r = i_{market} - \pi$$

This **real interest rate** represents the real return to lenders measured in terms of the purchasing power of interest paid. For example suppose we have the following:

A one year loan (**N** = 1) with the following terms:

Principal '**P**' = \$1000, *and*
nominal rate of interest '**i**' = 5%.

At the time the loan is made, the price of a common commodity 'Gasoline' (P_{gas}) is equal to \$1.00/gal. In real terms the lender is providing the borrower with the purchasing power equivalent to 1000 gallons of gasoline.

At the termination of the loan the borrower repays the principal '**P**' of \$1000 plus an interest payment '**I**' of \$50 (\$1000 x 0.05). If when the loan is repaid one year later, the price of gasoline P_{gas}' has risen to \$1.03/gal. (*a 3% rate of inflation*); the purchasing power of the principal plus interest (\$1050) will be equal to 1019 (\$1050/1.03) gallons of gasoline. In real terms, the purchasing power of the lender has increased by roughly 2%.

If the price of gasoline had risen to \$1.07 (*a 7% rate of inflation*) then the purchasing power of the repayment would have been equal to 981 (\$1050/\$1.07) gallons of gasoline. In this case the lender provided the opportunity for the borrower to acquire 1000 gallons of gasoline and at the termination of the loan the borrower repaid to the lender the ability to acquire only 981 gallons. An unexpectedly high rate of inflation had had an adverse impact on the lender -- a negative real rate of return.

If $E[\pi_t]$ is greater than π_t then 'r' will exceed 'r*' to the benefit of lenders (*real returns to lending greater than desired and perhaps greater than the rate of real economic growth*) as shown by the following operation. Substituting for i_{market} we have:

$$r = r^* + E[\pi\ ] - \pi$$

If $E[\pi\ ]$ is less than 'π' , then benefits will accrue to the borrower otherwise benefits accrue to the lender in the form of relatively high real interest rates.

During the 1980's, many economists felt that the real rate of interest was abnormally high (i.e., in excess of 2.5 – 3.0%). This may be explained in part due to the inflationary expectations that built up in the late 1970's and early 1980's. Nominal interest rates have taken these expectations into account. The effects of these inflationary expectations differing from the actual rate of inflation can be seen in the table below where the annualized 6-month T-bill rate is used as a measure of the market interest rate:

Table 2, Real Interest Rates and Inflation Rates

Year	T-Bill Rate	r* (desired)	$E[\pi_t]$	π_t (actual)	r (actual)
1978	7.57	4.5	3.07	9.0	-1.43
1980	11.37	2.0	9.37	12.5	-1.13
1982	11.08	1.5	9.58	3.8	7.28
1984	9.80	4.0	5.8	3.9	5.90
1986	6.03	3.5	2.53	1.1	4.93
1988	6.92	3.5	3.42	4.4	2.52
1990	7.47	1.2	6.27	6.1	1.37
1992	3.57	2.7	0.87	2.9	0.67
1994	4.66	3.5	1.16	2.7	1.96
1996	5.09	2.8	2.29	3.3	1.79
1998	4.85	4.4	0.45	1.6	3.25
2000	5.92	4.3	1.62	3.4	2.52

Source: Economic Report of the President 2002

Note, in the above table, the desired real rate of interest (r*) is based on an average of the actual rate of real economic growth over the previous three years (author's calculations).

The Term and Risk Structure of Interest Rates

By taking credit risk and the length of the lending period into account, the many different nominal interest rates at a point in time listed above in **table 1**, can be explained with the following equation:

$$i_{term,risk} = \{r^* + E[\pi_t)]\} + \rho + \lambda$$

The first two components in brackets are the familiar desired rate of return and expected inflation that make up the core of any interest rate at a point in time. The third component 'ρ' is known as the **risk premium** established by credit markets for different categories of risk. This value may be large or small depending on how risk-averse lenders might be at any point in time.

The last component 'λ' is known as the **liquidity premium** which represents the amount of compensation required by a lender for lending to the long end of the market. For example in the above table the T-Note rate is 6.25% and the T-Bond rate is 7.40%. The 1.15% difference implies that lenders require an additional $11.5 per $1000 lent for 30 year loans relative to 5-10 year loans to the Federal government. Greater uncertainty about future rates of inflation or future political events will often widen the spread between the medium and long term. The differences that exist in nominal rates due to this liquidity premium are summarized in the frequently published yield curve constructed by using the different treasury rates (risk-premium = 0) that exist on a given date.

The (Credit) Risk Premium
The No Risk category in **table 1** corresponds to Federal Government debt (T-bills, T-Notes, and T-Bonds). In this category, there is absolute certainty that the borrower (the Federal Government) will be able to properly service the debt and repay the principal at all times. This is possible because the Federal Government can always borrow new funds at what ever rate of interest necessary to pay existing interest obligations or to repay any existing debt. The government is not constrained by an income statement of annual profit and loss as are private companies. In addition, unlike state and local governments the Federal Government has the power to establish or perhaps create the currency necessary to meet its existing obligations.

The Low Risk category corresponds to a S&P classification of **AAA-AA** or investment grade lending. Borrowers in this category have a strong history of debt repayment and a solid stream or revenues to service any future debt. Lenders in this category are very risk averse seeking to protect their asset base (the principal) by avoiding those borrowers who might default on their debt repayment.

The classification of **A-BB** represents somewhat speculative grade lending or Medium Risk. Borrowers in this category often have a good credit history, however, there is some uncertainty about future revenues to service additional debt. Lenders involved in this type of debt are willing to speculate that all interest payments and principal repayment will take place in return for a slightly higher return on their investment.

Finally the High Risk category carries a S&P rating of **B-CCC** also known as "junk" or highly speculative lending. Lenders in this category are willing to put their principal at risk in return for a high return as measured by usually double-digit yields for a limited period of time. There is a strong probability of default on debt in this category.

The Liquidity Premium and Term Structure
Differing lengths in the lending period correspond to different degrees of uncertainty about future events. Very little change takes place in the political or economic structure of a nation or the world in any given year--the short-term. However, over a 30 year period of time typical for some types of government borrowing (T-Bonds) and private borrowing (home mortgages), massive changes may take place in rates of inflation, political conflict, and the global balance of power. Additionally holders of longer term debt instruments will find that the (secondary) market value of these assets is more sensitive to changes in market interest rates relative to shorter term instruments.

In the long-term tremendous uncertainty exists and yet there are institutional lenders that actively seek the long term. For example, pension funds and life insurance companies that need to plan for exact financial obligations well into the future.

Three widely discusses hypotheses exist describing the term structure of interest rates and determination of the liquidity premium. Two of these hypotheses, **Segmented Markets** and the **Expectations Hypothesis** represent extreme propositions with regard to the ability to substitute between short-term and long-term assets. The third hypothesis, **Preferred Habitat**, is an attempt to find some middle ground.

Segmented Markets: At one extreme is the notion that short term and long term assets are highly imperfect substitutes. In this approach the borrowers and lenders (sellers and buyers) in short term markets are very different individuals and agents from those who interact on long term markets. Short term borrowers include those who need to finance seasonal inventory or need to smooth out their cash position over a fiscal period. Lenders include those with cash surpluses who do not want to sacrifice much in the way of liquidity. The price (and thus the yield) on short term assets are determined by the interaction of these individuals in primary and secondary markets.

A different set of agents interact in the long term market. The borrowers in this end of the market are those financing homes in mortgage markets or seeking capital to build factories or other similar long term projects. Attempting to borrow short term would make it difficult to service such large loans relative to annual income or revenue. Lenders would include pension funds, insurance companies and others collecting current premiums but with well-identified long term obligations. Buying and selling between these agents determine long term asset prices and yields.

Differences in yields that may exist between the short term and long term are explained only though the forces of supply and demand in each market. Accordingly, long term yells could be greater than, equal to, or less than short term yields.

The Expectations Hypothesis: At the other extreme is the idea that short term and long term debt instruments are prefect substitutes for one-another. An important consideration here is that lenders have more flexibility with regards to the length of the lending period relative to borrowers. Many borrowers enter the long term market precisely because the nature of their project is long term. For these projects to be financially feasible the borrower needs to rely on a long continuous stream of revenues to repay the debt. Such projects are just not possible in a one to ten year horizon. Many home owners find that housing is affordable only if they can stretch the loan payments over a 30 year period of time given their annual income.

Lenders, however, have a choice. A lender can make a loan for 30 years or that individual can make six sequential five-year loans. The 30 year loan locks in an interest rate for the duration of the loan at the prevailing long-term rate whereas the sequence of six medium-term loans exposes the lender to changes in nominal rates each time the funds are reinvested. The long-term loan exposes the lender to the uncertainty of distant future events in contrast to the medium term sequence which allows the lender to react to changing economic conditions. There is a balancing act taking place between uncertainty about future economic conditions and the direction of future interest rates.

For example, suppose that a lender chooses to lend a principal sum '**P**' for two years. The agent has a choice of making a single loan at the present two-year rate '${}_2r_t$' or making a one year loan at the existing short-term (one-year) rate '${}_1r_t$' and reinvesting at the expected prevailing short-term rate in the following year '$E[{}_1r_{t+1}]$'. The notation presented '${}_kr_t$' tracks the term of a loan 'k' and the time period 't'. Thus for the lender:

$$\mathbf{P}(1 + {}_2r_t)^2 = \mathbf{P}(1 + {}_1r_t)(1 + E[{}_1r_{t+1}])$$

or

$$1 + 2({}_2r_t) = 1 + {}_1r_t + E[{}_1r_{t+1}]$$

or

$${}_2r_t = \{{}_1r_t + E[{}_1r_{t+1}]\}/2$$

This last expression states that the current two-year rate will be an average of the current one-year rate and the expected one-year rate next year. If the two year rate is greater than this average, the lender will invest in the longer-end of the market seeking the higher yield. This buying of the two-year instrument will drive its price upward and the two-year yield (interest rate) downward until both sides of the above expression are equal. If we generalize the last expression for an n-period interest rate we have:

$${}_nr_t = \{{}_1r_t + E[{}_1r_{t+1}] + E[{}_1r_{t+2}] + \ldots + E[{}_1r_{t+n}]\}/n$$

The long term rate and the liquidity attached to it is based on an average of current actual and expected future short-term rates.

Preferred Habitat: This third hypothesis is an attempt to marry the relevant features of the above two approaches. This hypothesis begins with the notion that lenders prefer the short term over the long term the latter exposing these agents to many types of future uncertainties. However, these agents can be induced into the long term via payment of a term-premium 'θ' over-and-above and average of current and expected future short-term rates:

$${}_nr_t = \{{}_1r_t + E[{}_1r_{t+1}] + E[{}_1r_{t+2}] + \ldots + E[{}_1r_{t+n}]\}/n + \theta$$

The actual derivation of liquidity and risk premiums take place in financial markets through the process of buying and selling financial instruments--concepts that will be discussed in the last section.

Be sure that you understand the following concepts and terms:

- Consumption Expenditure
- Investment Expenditure
- Government Expenditure
- Net Export Expenditure
- Marginal Propensity to Consume
- Private Savings
- Public Savings
- Foreign Savings
- National Savings
- Flow of Funds
- Inflation
- Inflationary Expectations
- Adaptive Expectations
- Consumer Price Index 'CPI'
- Nominal Interest Rate
- Real Interest Rate
- Desired Real Rate of Return
- Risk Premium
- Liquidity Premium
- Term Structure of Interest Rates
- Expectations Hypothesis
- Segmented Markets
- Preferred Habitat

Problem Set #2, Real Interest Rates

1. Values for Real GDP and the Consumer Price Index (CPI) are as follows:

	RGDP	CPI	π	r
1999	$8,675	166.6		
2000	$8,910	175.2	_______	______

Calculate the rate of inflation between 1999 & 2000, derive the real rate of interest (return) for the year 2000 if nominal interest rates are 7%. Is this real rate of interest above or below the rate of economic growth for the same period of time ? ___________ Is this to the benefit of lenders or borrowers?_______ Explain.

2. In any debt contract, both borrower and lender come to an agreement with respect to the nominal rate of interest based in part on inflationary expectations. Lenders include this inflation premium to protect the purchasing power of the funds being lent. Borrowers agree to this rate on interest because they expect that future inflation will enhance their ability of repay the debt. The ability to pay the debt is known as the (interest) burden of the debt and may be calculated as follows:

$$\text{Burden} = \frac{\text{Interest Expense}}{\text{Income}}$$

Assume that an individual borrows $100,000 to purchase a house at 7% interest. Embedded in this interest rate is an inflation premium (π^e) of 5%. Both borrower and lender agree to this inflation premium.

a. Given this information, what is the real rate of return of the loan to lenders?______

b. If the borrower has an annual income of $28,000, what is the debt burden of this loan?___________

c. If the actual rate of inflation (p) is 5% over the first three years of the loan, calculate the level of income (growing at this same rate) and the (interest) burden of the debt:

	Income (at 5% growth)	Burden	Income' (at 3% growth)	Burden'
Year 1:	_____________	_________	_____________	__________
Year 2:	_____________	_________	_____________	__________
Year 3:	_____________	_________	_____________	__________

What is happening to the burden of the debt over time?_______________________

d. Now perform the same calculations for an annual rate of inflation of 3%. Do borrowers benefit or are they hurt by this lower rate of inflation? ___________

Problem Set #2, page 2

3. The CPI, Inflation, and Real Interest Rates

a. You represent an average consumer such that your income is allocated among the following four goods in the corresponding proportions:

Gasoline (energy)	20%
Housing	40%
Food	30%
Clothing	10%
Total	100%

b. You are also a lender agreeing to the following lending terms:

Principal **P**	= $1000
Nominal Interest Rate .**i** = 10%	= 0.10
Term **N**	= 1 yr.

c. At the time the loan was made the following prices prevailed in the economy:

P_{gas}	$1.00 gal
$P_{housing}$	$100,000 house
P_{food}	$10.00 unit
$P_{clothing}$	$10.00 unit

d. Given the proportions of part (a), the dollar amount of the loan in part (b) and the prices of part (c), how much Gasoline, Housing, Food, and Clothing are you lending to the borrower?

e. At the time the loan in repaid, the following prices prevailed:

P_{gas}	$1.05 gal
$P_{housing}$	$102,000 house
P_{food}	$12.00 unit
$P_{clothing}$	$10.00 unit

f. How much Gasoline, Housing, Food, and Clothing are being repaid to you? Are you better or worse off upon repayment of the loan? Explain

g. Calculate a value for the CPI using parts (a) & (c) as base-period values and determine the annual rate of inflation over the term of the loan.

h. What is the real rate of return? Using this data, are you better or worse off upon repayment of the loan? Explain.

Problem Set #3: Savings and Investment

1. Potential Output **'Y*'** for a given economy is $10,000 [i.e., $10 trillion]. Given the following equations:

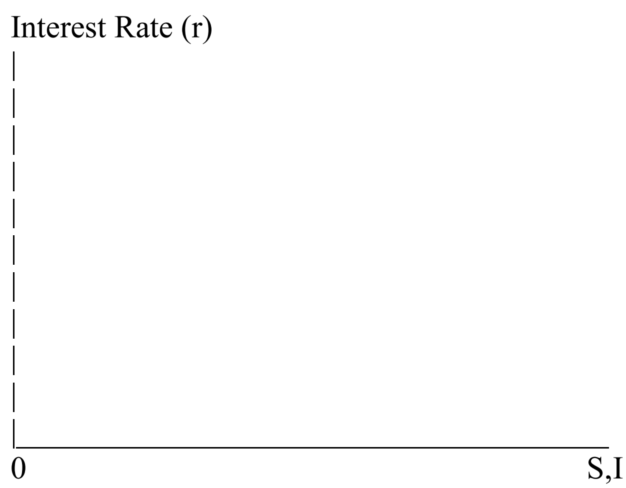
$C = 0.80(\mathbf{Y^*} - T)$	-- Consumption Expenditure
$T = 0.10\mathbf{Y^*}$	**--** Taxes [*tax rate* = 10%]
$G = \$2,000$	-- Government Expenditure
$I = \$1,500 – 100(r)$	-- Domestic Investment Expenditure [r = *market interest rate*]
$NX = 0$	-- Exports = Imports

a. Calculate the following:

- Private Savings,
- Public Savings, and
- National Savings.

b. At what market interest rate will Domestic Investment be equal to National Savings?

Graph the results of parts 'a-c' in the diagram below:



c. Describe how an increase in the tax rate from 10% to 15% will affect Private, Public and National Savings and the market rate of interest.

Problem Set #3, page 2

2. Given the following:

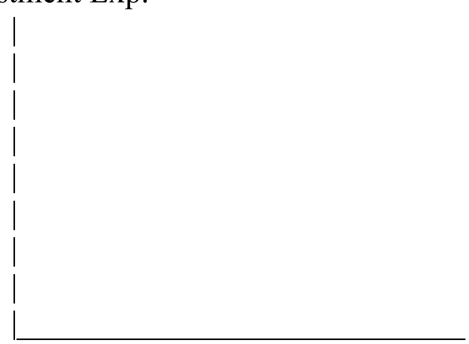
$Y^* = \$10,000$	--	Potential Output (held constant)
$C = b(Y^* - T)$	--	Consumption Expenditure [b = Marginal Propensity to Consume]
$T = 0.10Y^*$	--	Tax Revenue [Tax rate = 10%]
$S_{public} = T – G = \$0$	--	Public Savings [G = T *always*!]
$S_{private} = Y^* - T – C$	--	Private Savings
$S_{national} = S_{public} + S_{private}$		
$NX = \$0$	--	Net Export Expenditure [always in balance]
$I_{domestic} = 1000 – 100(r)$		Investment Expenditure [r = interest rate]
$I_{domestic} = S_{national}$	--	Assume that the interest rate adjusts such that Domestic Investment Expenditure is always equal to National Savings.

Complete the following table:

MPC	Potential Output	=	Consumption Expenditure	+	Government Expenditure	+	Investment Expenditure	National Savings	Interest Rate
0.50	$10,000								
0.55	$10,000								
0.60	$10,000								
0.65	$10,000								
0.70	$10,000								
0.75	$10,000								
0.80	$10,000								
0.85	$10,000								
0.90	$10,000								
0.95	$10,000								
1.00	$10,000								

and Graph the relationship between *Consumption Expenditure* and *Investment Expenditure* in the diagram to the right:

Investment Exp.



Investment Decisions

Investment represents the acquisition of scarce resources in support of adding to the capital stock in production processes. The investment relationship may be written as follows:

$$\mathbf{J}_t = K^*_t - K_{t-1} + \delta K_{t-1}$$

where

- $\mathbf{J}_t$ = *Gross Investment (measured in units of new or replacement capital),*
- K^*_t = *Desired (profit-maximizing level) Capital Stock,*
- K_{t-1} = *Existing Capital Stock,*
- δ = *rate of depreciation*.

The difference in the first two terms ($K^*_t - K_{t-1}$) represents **net or new investment** and the last term (δK_{t-1}) represents **replacement investment**. **Gross investment** is just the sum of the two expressions.

By multiplying both sides of the equation by P_k (*the price of capital*) and factoring out 'K_{t-1}', we can write an expression for **Investment expenditure** ($\mathbf{I}_t$):

$$\mathbf{I}_t = P_k[K^*_t - (1-\delta)K_{t-1}]$$

where $\mathbf{I}_t = P_k\mathbf{J}_t$.

Profit Maximizing Behavior: The Net Present Value Approach

The key to understanding investment decisions is the determination of K*, the **desired capital stock**. This desired level is based, as stated above, on profit-maximizing behavior of the firm. There are several methods by which we can approach the profit-maximizing level of capital stock.

First, we can analyze the **net present value** of different amounts of capital to determine which quantity gives the greatest discounted stream of profits. For example, given the following:

- P_k = \$1000/unit,
- δ = 10%/year which implies that a unit of capital has a life of 10 years,
  We assume that the capital has no salvage value,
- r = 5%/annually -- this is the real market rate of interest,
- P_x = \$5 -- the market price of a unit of output.

With this data we will use the following annuity factor computation:

Note: The **Present Value** of an Income producing asset that generates an annual income stream (revenue) of '**R**' is defined by the following formula:

$$\mathbf{PV}_{asset} = \Sigma_{[t=1,N]} \mathbf{R}_t(1+r)^{-t}$$

or using the formula for a sum of a geometric series:

$$= (\mathbf{R}/r)[1 - (1+r)^{-N}]$$

as $N \to \infty$, this expression reduces to R/r.

$$PV_N = (\text{Revenue})[1 - (1+r)^{-N}] \div r$$

$$= (\text{Revenue})[1 - (1.05)^{-10}] \div 0.05$$

$$= (\text{Revenue})7.722$$

Table 3, Net Present Value

Input K	**Output (units 'X")**	**Revenue [P_x(X)]**	**Present Value (N=10)**	**Costs [P_k(K)]**	**Net Present Value**
1	50	$250	$1930.50	$1000	$930.50
2	90	$450	$3474.90	$2000	$1474.90
3	**120**	**$600**	**$4632.00**	**$3000**	**$1632.00**
4	140	$700	$5405.40	$4000	$1405.40
5	150	$750	$5791.50	$5000	$791.50

In the above table we have a short run production relationship with capital being the only variable input. The input-output relationship is derived under assumption of **diminishing marginal productivity**. We find that three units of capital would generate the greatest net present value and thus the greatest profits.

Profit Maximizing Behavior: The Marginal Revenue Product Approach
A second approach is the marginal approach where we compare the contribution to revenue by using one more unit of capital with the costs of acquiring that unit of capital. The contribution to revenue is known as the **Marginal Revenue Product** of capital and is calculated by multiplying the marginal product with the market price of the output produced:

$$\mathbf{MRP}_k = MP_K P_x$$

The contribution to costs is known as the **Rental Cost of Capita**l that includes borrowing costs (*or opportunity costs of using internal funds for investment expenditure*) and depreciation costs:

$$\mathbf{RCC} = P_K(r) + P_K(\delta) = P_K(r + \delta)$$

The desired level of capital stock **K*** is that quantity where:

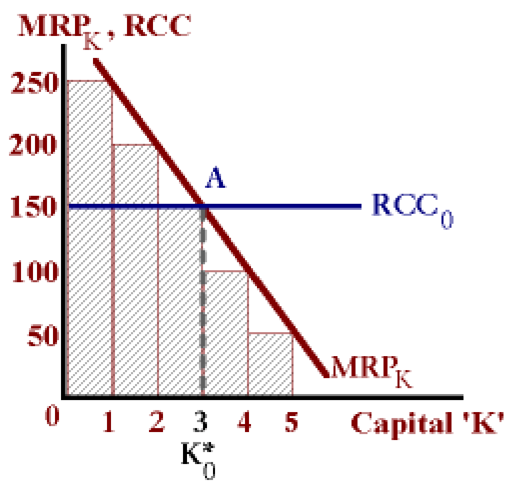
$$MRP_K = RCC$$

We will use similar data and a table for an example:

Table 4, MRP, RCC

K	**Output**	**MP$_k$**	**MRP$_k$**		**RCC (r = 5%, δ=10%)**
1	50 units	50	$250		$150
2	90	40	$200		$150
3	**120**	**30**	**$150**	**=**	**$150**
4	140	20	$100		$150
5	150	10	$50		$150

Figure 5, MRP, RCC



If the **MRP** exceeds **RCC** then profits will increase by acquiring additional units of capital (*contribution to revenue exceed contribution to costs*). If the opposite is true then the additional costs associated with one more unit of capital exceed the revenue generated and profits will decline. These relationships are shown in the diagram to the right:

See: The Digital Economist: http://www.digitaleconomist.com/capital.html *to experiment with changes the in relevant parameters of this model.*

Similar to the **Net Present Value** approach, we find that with three units of capital the contribution to revenue of this third unit is just equal to the costs of acquiring and using that third unit--profits will be a maximum at this level of input.

Present Value of a Perpetuity Approach

A third approach is using a calculation similar to the present value of a perpetuity. We begin by using the marginal conditions above:

$$MRP = RCC$$

$$MP_K P_x = P_K(r + \delta)$$

And we rearrange the terms:

$$[MRP_K - P_K(\delta)] \div P_K = \text{'}\Psi\text{'}$$

This term ‘Ψ' represents the yield on the last unit of capital employed in the production process:

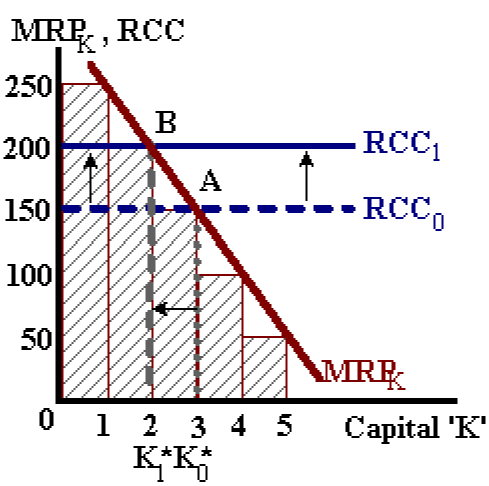
Table 5, Present Value of a Perpetuity

K	**Output**	**$MRP_K - P_K(\delta)$**	**P_K**	**Yield**	**r_{market}**
1	50 units	\$250-\$100	\$1000	15%	5%
2	90	\$200-\$100	\$1000	10%	5%
3	**120**	**\$150-\$100**	**\$1000**	**5%**	**5%**
4	140	\$100-\$100	\$1000	0%	5%
5	150	\$50-\$100	\$1000	NA	5%

In this case, we find that the yield on the first two units of capital is greater than the market interest rate ‘r_{market}‘ and thus may be acquired to earn profits over-and-above the borrowing costs. It is the third unit of capital where the yield is just equal to the market rate of interest -- no additional profits may be earned by hiring additional capital.

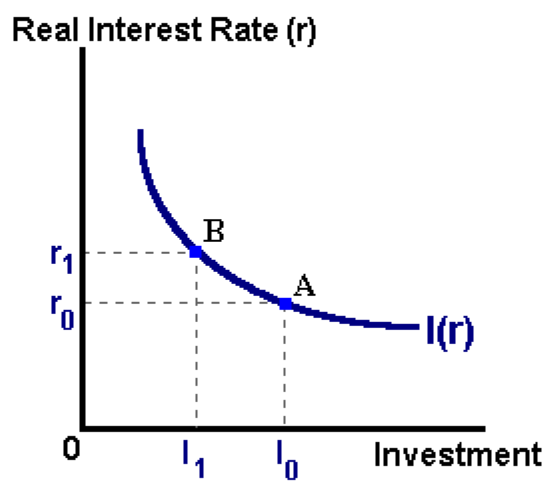
In all three approaches we find that the desired capital stock **'K*'** is equal to three units based on the productivity of capital, the rate of depreciation, the price of capital and the output being produced and, of greatest importance, market interest rates. An increase in the rate of interest will increase the Rental Cost of Capital ‘RCC’ and reduce the value of K*. Investment expenditure will decline as well as shown in the diagrams below:

Figures 5a, The Optimal Capital Stock



as **r**↑, **RCC**↑ and **K*** ↓

Figure 5b, Investment Demand



as **r**↑ , **I** ↓

We can use a Cobb-Douglas form of the production function to solve for K* and combine this with the expression for investment defined above:

$X = \mathbf{A}L^{\alpha}K^{\beta}$ *and* $MP_K = \beta X / K$

given the profit-maximizing condition:

$\mathbf{MRP_K} = \mathbf{RCC}$,

$\boldsymbol{\beta}\mathbf{(X/K)P_x} = \mathbf{P_K(r + \delta)}$

or

$\mathbf{K^*} = [\beta(P_x)X] \div [P_K(r + \delta)]$

Inserting this expression for K* into the investment expenditure equation gives us:

$\mathbf{I}_t = P_K\{ [\beta(P_x)X] \div [P_K(r + \delta)] - (1\text{-}\delta)K_{t-1}\}$

or

$\mathbf{I}_t = [\beta(P_x)X] \div (r + \delta) - (1\text{-}\delta)P_K K_{t-1}$

where we find that Investment expenditure is <u>inversely related</u> to market interest rates '**r**' and the price of a unit of capital 'P_K', <u>positively related</u> to output prices 'P_x' and the productivity of capital (*as measured by the parameter 'β'*), and <u>undetermined</u> with respect to the rate of depreciation 'δ'.

Be sure that you understand the following concepts and terms:

- Marginal Productivity
- Marginal Revenue Product
- Rental Cost of Capital
- Real Rate of Interest
- Rate of Depreciation
- Gross Investment
- Net Investment
- Replacement Investment
- Desired Capital Stock
- Present Value
- Present Value of a Perpetuity

Problem Set #4: Capital Accumulation and Investment

1. Given the following data:

Capital [**K**]	Output [**X**]	MP_K	Output Price (P_x)	MRP_K (MP_K x P_x)	Output Price (P_x')	MRP_K'
50	1000	70	$2.00	$140	$4.00	_______
60	1600	60	2.00	120	4.00	_______
70	2100	50	2.00	100	4.00	_______
80	2500	______	2.00	_______	4.00	_______
90	2800	______	2.00	_______	4.00	_______
100	3000	______	2.00	_______	4.00	_______
110	3100	______	2.00	_______	4.00	_______
120	3150	______	2.00	_______	4.00	_______

- The Price of a unit of capital (P_k) is $320
- The [expected] life of a unit of capital is 10 years
- The Optimal level of the capital stock '**K***' is defined where:

$$MRP_K = P_K(r + \delta)$$

- 'δ' represents the annual rate of depreciation

Complete steps a – d for a market output price 'P_x' of $2.00

a. If the real rate of interest 'r' is 15% (0.15), what is the optimal level of capital stock '**K***"?

b. If the real rate if interest falls to 2.5%, what is the new optimal level of capital?

- Investment is defined as: $I_t = K^*_t - K_{t-1} + \delta K_{t-1}$

c. If K_{t-1} is equal to 70 units of capital, what will be the level of *investment expenditure* for r = 15% at the existing rate of depreciation?

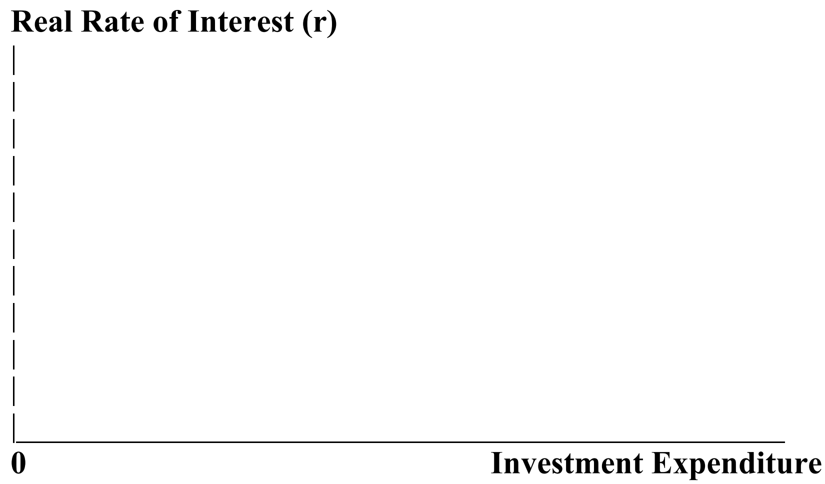
d. What will be the level of *investment expenditure* for K_{t-1} = 70, r = 15% and a rate of depreciation 'δ' of 0.225?

e. Repeat steps 'a-c' for an output price (P_x') of $4.00.

Problem Set #4: page 2

(Question #1, cont.)

f. Graphically show the relationship between investment expenditure and the real rate of interest. Specifically, show the level of expenditure for $K_{t-1} = 70$, $\delta = 0.10$, $P_x = \$2.00$, $r_0 = 15\%$, and $r_1 = 8.75\%$.



2. Suppose that production is defined by a Cobb-Douglas production function

$$Y = A\mathbf{L}^{1-\alpha}\,\mathbf{K}^{\alpha} \qquad \alpha = 0.30,\ \mathbf{A} = 100$$

a. Provide an economic interpretation for the parameters ‘α’ and ‘**A**’.

b. Does this production function exhibit *increasing returns*, *constant returns*, or *decreasing returns* to scale?____________________________ Explain.

Using the following expression for the *Rental Costs of Capital* (RCC), solve for the optimal level of capital stock ‘**K***’ as a function of the real interest rate ‘r’:

$$RCC = P_k(r + \delta) \qquad = \$320(r + 0.10)$$

Remember that K is that level of capital where* $RCC = MRP_K$.

c. By how much does the optimal capital stock change when the interest increases from ‘r = 10%’ to ‘r = 8%’?