WEALTH, CAPITAL ACCUMULATION and LIVING STANDARDS

Imagine a country where the primary goal of its economic policy is to accumulate a single commodity -- gold for example. Does the accumulation of wealth in this manner generate benefits to the members of this economy? Yes, but only if another country exists that devotes its energy and resources to the production of food, clothing, and other essentials and that this second country is willing to trade these goods for the gold of our first country.

Individuals cannot directly consume commodity wealth. Gold, oil, iron ore, and the like provide no nutrition or protection from the elements. These commodities have little value in direct consumption. However, if trade is possible with another nation -- a nation that engages in production of consumable goods and services, then these commodities do have value.

Adam Smith was the first to realize that the *Wealth of a Nation* is not in the accumulation of commodities, nor in the resource reserves that a nation may happen to possess. But rather wealth exists in the productive knowledge of its people. The ability to efficiently transform resources (factor inputs) into desired goods and services represents the true source of a nation's wealth.

Physical and human capital represents the true source of wealth. This wealth is used to generate factor income as a payment for the production of desired goods and services X_i [i = 1,2,...,n-goods] and income to be used to purchase these same goods and services. Thus wealth '**W**' may be measured in terms of the future stream of income (real goods and services) 'Y_t', discounted at some rate 'ρ', generated by the use of physical and human capital:

$$X_i = f(L_i, K_i, M_i) \qquad \textit{for all } i = 1,2,...n\textit{-goods}$$

and

$$Y_t = \Sigma_{[i=1,n]} P_{i,0}\, X_{i,t}$$

(*pre-multiplying by current market prices allow for aggregation*) such that:

$$\mathbf{W} = \Sigma_{[t=1,T]} \frac{\Sigma_{[i=1,n]} P_{i,0}\, f(L_{i,t}, K_{i,t}, M_{i,t})}{(1+\rho)^t} \qquad \textit{A monetary measure of wealth.}$$

This rate of discount 'ρ' is a measure of how individuals discount future economic activity (and use of resources) relative to the present and can act as a proxy for attitudes about uncertainty about the economic future and political stability. Via use of this rate we then are able to convert the stream of present and future income/output -- *a flow variable* into a measure of wealth -- *a stock variable*.

The numeric value of wealth is really less important than what it represents. The individual members of a given society are interested in living standards such that growth in output will, at a minimum, equal or exceed the rate of population growth. Thus they are interested in a stock of human and physical capital sufficient to produce this desired growth in income.

Note: Over time labor input will grow at some rate 'n' proportional or perhaps equal to the rate of population growth:

$$L_{,t} = L_0(1+n)^t$$

Growth in capital is affected by savings rates in a given economy to support gross investment (*changes in the capital stock overtime*) at some rate 'g' less the rate in depreciation 'd'. This rate of depreciation is a reflection of the fact that, over time, capital does where out. Thus 'g - d' represents the net growth in the capital stock over time.

$$K_t = K_0(1 + g - d)^t$$

Increases in living standards require more capital be made available per unit of labor thus making each unit of labor more productive or:

$$(g - d) > n$$

such that:

K_t/L_t is increasing over time.

To present several examples of production and its relationship to wealth, we will examine the production of a simple restaurant meal, passenger services on an airplane, and production of housing services.

A sole proprietor operating a small restaurant uses raw materials in the form of food ingredients, capital in the form of a stove or oven, and labor input in the form of his own time. The proprietor is motivated by the fact that he is able to create a meal that is valued by his customers over-and-above the value of the individual inputs. This added value is created by his talents and know-how as a cook or chief combined with the physical capital of the restaurant. Wealth in this case is not only in the materials or factors of production but also in the proprietor's knowledge of preparing a meal. Over time this human capital will generate a stream of income for the proprietor for as long as he operates the restaurant and as long a there is a demand for his product.

An airline makes decisions to purchase an airplane based on anticipated demand for transportation services. The value of these services are based on the benefits customers receive by flying relative to other forms of traveling from one place to another. Like the restaurant the value of the airplane (a unit of physical capital) is in the provision of passenger services. Building this airplane represents an addition to wealth in that it will generate a stream of revenue (income) for the corporation generated over its physical life provided demand for passenger services remain.

Finally, the construction of an apartment building represents an addition to wealth based on the demand for apartments by those seeking housing services. The building will generate a stream of rental income over its life based on occupancy levels and rental rates. If however, the building is largely vacant--demand is lacking, its contribution to (national) wealth is close to zero. It becomes an asset with limited realized value even though its construction represented a combination of valuable materials, labor services, and land area.

The creation of wealth is based on knowledge -- the ability to take raw inputs and convert them into output with value greater than the sum of the individual parts. Additionally, this value is determined by correctly assessing the demand for the output -- how it will satisfy needs and wants. Creation of a restaurant, airplane, or apartment building (*physical capital*) all represent a contribution to a nation's wealth in that they all generate a future stream of income based on the willingness of the members of that nation to purchase food services, transportation services, or housing services to satisfy specific wants. Creation of a school teacher or engineer (*human capital*) also represent additions to a nation's wealth in that they also generate services desired by others in a given economy and thus produce a stream of income for the individual based on demand for those services.

Living Standards

If we define the **Standard of Living 'SoL'** as the ratio of Real GDP and the population of a given country (*also known as per-capita Real GDP*), then improvement requires that this ratio increase over time:

$$\mathbf{SoL} = \text{RGDP / Population}$$

Such that:

$$\%\Delta\text{RGDP} > \%\Delta\text{population} \quad \Rightarrow \quad \mathbf{SoL} \uparrow$$

In a world of diminishing marginal productivity, this increase can be difficult to achieve.

Let us take a look at a basic production relationship. In this case, we will hypothesize an aggregate production function defined as follows:

$$\mathbf{Y^*} = f(L, K, M)$$

Where **Y*** represents the output of an economy (RGDP), and 'L', 'K', and 'M' represent the aggregate factors of production: Labor, Capital, and Materials. Looking at production in the short run, holding 'K' and 'M' constant we define the following hypothetical relationship:

$$\mathbf{Y^*} = f(L) = 100(L)^{0.50}$$

Table 1, An Aggregate Production Function

L	**Y***	**MP_L**	**Y/L**
0	0	-	-
1	100	100	100
4	200	33.3	50
9	300	20.0	33.3
16	400	14.7	25
25	500	11.1	20

We observe that as the amount of labor increases, output increases at a decreasing rate (*diminishing marginal productivity*). Also notice that as more labor is used in the short run, the ratio between RGDP '**Y***' and **L** is also falling.

Now suppose that the ratio between the population and the size of the labor force (*the **labor force participation rate***) remains constant and equal to 0.67.

$$L = \alpha(Pop) = 0.67(Pop)$$

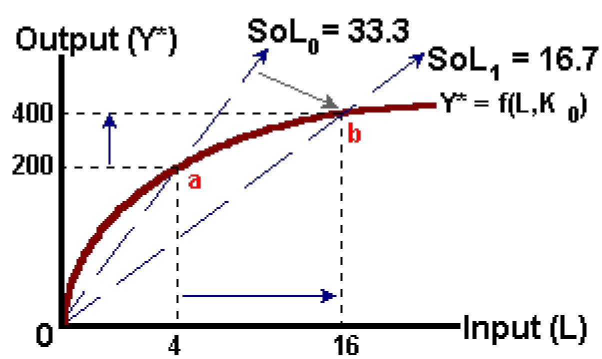
We can derive an expression for the Standard of Living (**SoL**):

$$\mathbf{SoL} = \mathbf{Y^*}/PoP = \alpha\,(\mathbf{Y^*}/L)$$

Or as more labor is applied to the production of goods and services the **SoL** declines:

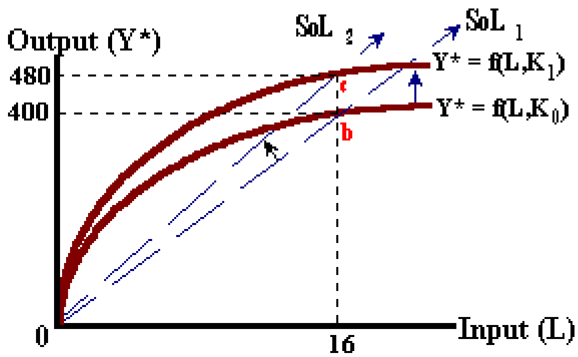
Figure 1, An Increase in Labor Input

L	**Y*/L**	**SoL**
0	-	-
1	100	67.0
4	**50**	**33.3**
9	33.3	20.0
16	**25**	**16.7**
25	20	12.6



In a country with population growth and diminishing marginal productivity what is necessary for improvements to living standards are additions to the capital stock, the level of technology or both. These additions increase the productivity of workers and allow for more output for each and every level of labor input. This can be seen in the diagram below:

Figure 2, An Increase in the Capital Stock $K_0 \rightarrow K_1$



Physical and Human Capital may be defined as **Intermediate Goods** or a *goods used to produce other goods* and represents production not intended for direct consumption. Rather, the creation and accumulation of capital is intended to increase the level of productivity of a nation and thus allow for an increase in the production of goods and services at future date.

The creation and accumulation of capital depends on the ability of a nation to give up current consumption of **Final Goods** (*goods used for direct consumption*) to make resources available for the accumulation of this capital. Deferring consumption (*known as savings*) depends on the ability of that nation to first meet the basic needs of its citizens with existing production technology and resource availability.

In every economy, a tradeoff always exists between using resources for the production of **Intermediate Goods** and **Final Goods** although the consequences of this tradeoff may differ among nations

For example, looking at the table below:

Table 2, Production Possibilities (*Final Goods vs. Intermediate Goods*)

	L	$Y_{(wheat)}$	MP_L		L	$K_{(plows)}$	MP_L
F	0	0	-	**A**	0	0	-
E	1	80 bu	80	**B**	1	50	50
D	2	150	70	**C**	2	90	40
C	3	210	60	**D**	3	120	30
B	4	260	50	**E**	4	140	20
A	5	300	40	**F**	5	150	10
	6	330	30		6	150	0
	7	350	20				

As labor is transferred from the production of one type of good to the other, more and more final goods (*wheat*) must be sacrificed to produce additional units of capital (*plows*). This tradeoff is defined by the **Production Possibilities Frontier** (PPF) in the diagram below (left). These sacrifices might be possible for some countries, but for those nations living at the edge of subsistence (*barely able to produce enough calories to support the population*), this reallocation might come at the expense of starvation for some. It is this tragic tradeoff that has contributed to Economics being labeled at the *Dismal Science*.

Figure 3a, Final vs. Capital Goods

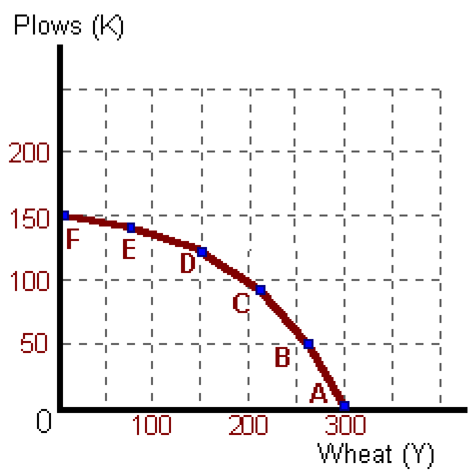
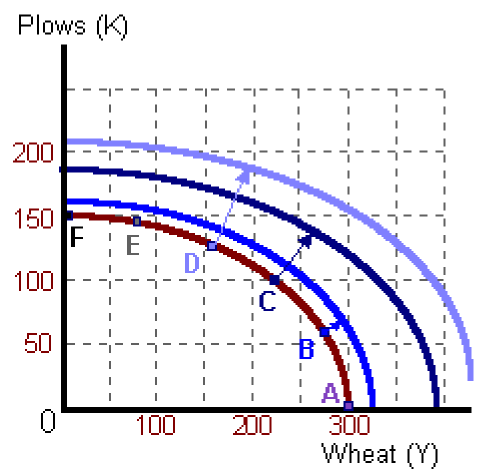


Figure 3b, Capital and Future Growth



Looking at the Figure 3b, we find that when resources are devoted to the production of 260 units of wheat and 50 plows (point '**B**'), some future growth is possible (the **blue** PPF). By using more resources in the production of plows (at the expense of labor, i.e. at point '**C**'), we see that a greater level of future growth is possible (the **navy** PPF). The additional plows manufactured in the present allows for labor to be more productive in the future such less labor could be used to produce much more wheat and remaining workers devoted to additional plow production.

Production

In order to understand the creation of wealth and the engine for economic growth that will provide for increasing standards of living, we must first start with an understanding of several characteristics govern aggregate production relationships.

First, is the law of **diminishing marginal productivity**. Holding other factors of production constant (*i.e., capital and/or materials*), increasing quantities of a single input lead to less and less additional output. This property is just an acknowledgement that it is impossible to produce an infinite level of output when some factors of production (*machines or land*) fixed in quantity.

Second, is that all factors are **essential in production** and to some degree one factor may be substituted for another factor of production. Increasing the amount of capital or machinery can replace some labor but not all of the labor in a production process. Increasing amounts of labor (greater care being taken in production to avoid waste) can reduce the need for some material inputs.

Third is that, for production in the aggregate, there are **constant returns to scale**. This property refers of the ability to double output simply by doubling the quantity of all the available inputs. A simple way to understand this is based on the idea that any production process may be replicated. Thus if a certain quantity of grain is being produced on one acre of land with 5 units of labor input and 3 pieces of capital, then by replicating this production process the quantity of grain produced may be doubled.

Note: One specific mathematical relationship that possesses these three properties is the **Cobb-Douglas** production function. This particular representation is one of several possibilities and may be written as follows:

$$X_i = \mathbf{A}_t L^{\alpha} K^{\beta} M^{\gamma}$$

where L, K, M represent the factor inputs listed above, **A** represents a measure of technology at time period 't', and the exponents represent production parameters (actually output elasticities) for each factor input. The fact that it is multiplicative in the inputs reflects the notion that one factor may be substituted for another. **Diminishing marginal productivity** requires that the exponents α, β, and γ each take on values less than one. Each input being **essential** and making a positive contribution to output implies that these exponents be strictly greater than zero. Finally, **constant returns to scale** implies that α, β, and γ sum to one.

Profit Maximizing Behavior

We also need to examine the behavior of those involved in the production process. One assumption is that of profit-maximizing behavior in that economic agents attempt to maximize the difference between the revenue from the sale of a particular good or service and the costs of production (w, r, & z represent factor prices). We can write the following:

$$\textit{max}\ \pi = R - C$$

or

$$\max \pi = P_i X_i - [wL + rK + zM]$$

s.t

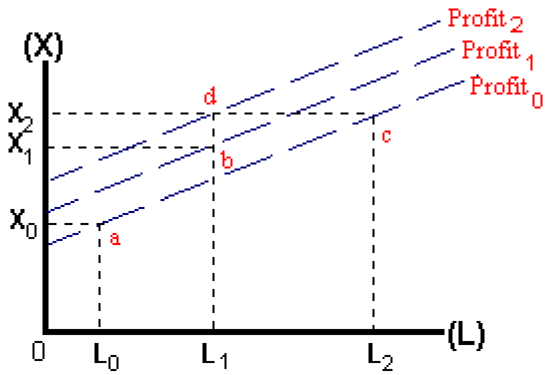
$$X_i = f(L,K,M).$$

The objective function can be rewritten by solving for the variable 'X' as follows:

$$X_i = [(\pi + FC)/P] + (w/P)L$$

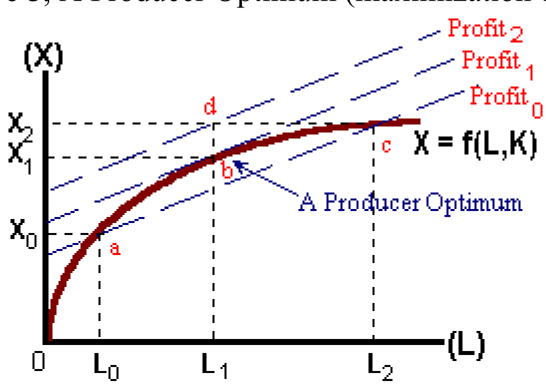
where FC represents the fixed costs of production (rK + zM). This expression is known as an **iso-profit function** with the term in the brackets being the intercept which represents a given level of profits and the term (w/P)-- also known as the real wage rate, represents the slope of this function. Any point on a particular line represents a given level of profits. For example, in the diagram below:

Figure 4, Iso-Profit Lines



The combination of L_0, X_0 corresponds to a level of profits of Profit_0. Likewise the combination of L_2 (*greater costs*) and X_2 (*more revenue*) also corresponds to this same level of profits (Profit_0) -- revenue and costs have increased by the same amount. However, the combination of L_1 and X_1 correspond to a greater level of profits relative to the combination of L_0, X_0 (*revenue increases more than costs*).

Figure 5, A Producer Optimum (maximization of profits)



By adding the **production function** to the above diagram, we find that the input- output combinations as defined by points 'a', 'b', and 'c' are all within the limits of available technology. Point 'd' however, is unattainable -- a level of output of X_2 is impossible with a level of labor input of L_1.

At point 'b', we find that we achieve the greatest level of profits possible with this existing level of technology. At this point, the production function is just tangent to iso-profit line 'Profit_1. Stated differently, the slope of the production function (**MP_L**) is just equal to the slope of the iso-profit line (**w/P**). This point is known as a producer optimum defined by the condition:

$$MP_L = w/P$$

See: The Digital Economist: http://www.digitaleconomist.com/po_tutorial.html
To practice with the parameters of this profit-maximization model.

Mathematically, we can solve for this producer optimum by substitution of the constraint into the objective function,

$$max\ \pi = P_x[f(L,K,M)] - wL - rK - zM$$

and solving (first-order conditions) with respect to any of the factor inputs we can derive the following:

$$d\pi/dL = P_x MP_L - w = 0, \quad d\pi/dK = P_x MP_K - r = 0, \quad d\pi/dM = P_x MP_M - z = 0$$

These results may be interpreted in a variety of ways.

First we could write (*using labor*) $\mathbf{P_x MP_L = w}$**,** where the left-hand side represents the **marginal revenue product of labor** -- that is the contribution of one more unit of labor input to the firm's revenue. The right-hand side '**w**' represents the prevailing **nominal wage rate** or the cost of hiring that additional unit of labor. Profit maximization implies that the contribution to revenue (*using monetary units of measure*) is just equal to the contribution to costs in the hiring of one more unit of a factor input.

Second, we can rearrange the terms and write $\mathbf{MP_L = w/P_x}$. This expression states that profit maximization implies that the contribution of each additional unit of labor input to output must be paid a **real wage** (*a measure of purchasing power*) equivalent to that level of contribution.

Third, we could write $\mathbf{P_x = w/MP_L}$ or $\mathbf{P_x = MC}$ which states that profit maximization occurs when the revenue from selling one more unit (*marginal revenue*) of a particular good 'Px' is just equal to the cost of producing one more unit (*marginal cost*) of that same good.

Note:	MC	$= \Delta$Total Costs/ΔX
		$= \Delta$Variable Costs/ΔX
		$= \Delta(wL)/\Delta X$
		$= w(\Delta L/\Delta X) = w/(\Delta X/\Delta L)$
	MC	$= (w/MP_L)$

The Contribution of Factor Inputs to Economic Growth

From our look at national income accounting, we observe that for the U.S. economy (based on the income approach), that labor income makes up roughly 70% of national income and non-labor income (proprietor's income, net rental income, corporate income, and net interest income) makes up the remaining 30% of national income. This can be written as follows:

NGDP $= PY = wL + rK,$

$wL = 0.70PY$, *and* $rK = 0.30PY$

Using the Cobb- Douglas production function with 'L' and 'K' representing labor input and non-labor input respectively and given constant returns to scale ($1-\alpha = \beta$), we can write:

$Y^* = \mathbf{A}L^{\alpha}K^{1-\alpha}$, -- *note*: 'Y*' (*Potential Output*) = 'Y' (*Real GDP*)

$MP_L = \alpha(Y^*/L)$ and

$MP_K = (1-\alpha)(Y^*/K)$

Note that the variable 'M' representing materials has been dropped under the assumption that labor and capital are combined in the extraction of these raw materials and the above factor payment percentages represents compensation for these types of activities.

With profit maximizing behavior,

$P_xMP_L - w = 0$ *or* $\mathbf{MP_L = w/P_x}$

and similarly

$\mathbf{MP_K = r/P_x}$

Therefore

$\alpha(Y^*/L) = w/P_x$ *or* $\boldsymbol{\alpha = wL / P_xY^*}$

and

$(1-\alpha)(Y^*/K) = r/P_x$ *or* $\mathbf{(1-\boldsymbol{\alpha}) = rK / P_xY^*}$

which implies that:

$\boldsymbol{\alpha = 0.70}$ and $\mathbf{(1-\boldsymbol{\alpha}) = 0.30}$

and

$\mathbf{Y^* = A_tL^{0.70}K^{0.30}}$

It can be shown that:

$dY^* = [\Delta Y^*/\Delta A_t]dA + MP_LdL + MP_KdK$

$dY^* = [L^{\alpha}K^{(1-\alpha)}]dA + [\alpha Y^*/L]dL + [(1-\alpha)Y^*/K]dK$

and dividing through by 'Y*', we can write (noting that for any variable 'x', $dx/x = \%\Delta x$):

$\%\Delta Y^* = \%\Delta A_t + \alpha[\%\Delta L] + (1-\alpha)[\%\Delta K]$

Expressed differently, the rate of economic growth (*assuming that it matches the growth rate in potential output -- an assumption that is valid over the long term*) may be expressed as

$$\%\Delta RGDP = \%\Delta Y^* = \%\Delta A_t + 0.70[\%\Delta L] + 0.30[\%\Delta K]$$

Economic growth is thus the sum of the rate of growth in technology in addition to a weighted average of the rate of population growth and the rate in which capital accumulates. An interesting implication of this is that, holding other factors constant, a population growth rate of 1% leads to a less than one percent growth rate in output--a decline in the **Standard of Living**. In order to maintain or improve these Living Standards, there must be an accumulation of capital and/or technological progress.

The Solow Growth Model

One way to understand the relationship between current production, savings activity and the accumulation of capital is via the **Solow Growth Model** which defines the tendency of different nations to approach an equilibrium (*steady-state*) level of the capital stock.

We begin by using the same economy-wide production function in Cobb-Douglas form with constant returns to scale as used above:

$$\mathbf{Y}^* = f(L,K) = \mathbf{A}L^{\alpha}K^{1-\alpha}$$

As a first step, we modify this expression to put it into a form that represents the **Standard of Living** (*simply the ratio of output to labor input*):

$$\mathbf{y}^* = \mathbf{Y}^*/L = f(1,K/L) = \mathbf{A}(K/L)^{1-\alpha} = \mathbf{Ak}^{1-\alpha}.$$

The term '**k**' represents the capital/labor ratio better understood as the amount of capital available per unit of labor input. We would expect that greater amounts of capital per labor-unit would make that labor more productive and thus raise the living standards within a particular nation.

Capital is unique in that over time it wears out. This factor input is subject to the effects of friction, obsolescence and climate such that at some future date it ceases to make a useful contribution to the production process. This is known as depreciation. The reciprocal of the life-span of a unit of capital is then defined as the rate of depreciation '**d**'. The implication of this is that without replacement (*via investment expenditure*) the capital-labor ratio would naturally decline over time. In order to maintain this ratio, the required level of investment '$I_{required}$' (*a flow variable*) must, at a minimum, be equal to the depreciation in the capital stock:

$$I_{required} = \mathbf{d}K$$

It must be noted that with greater amounts of capital in place, greater levels of investment are required to maintain a particular capital-labor ratio (i.e., *the more capital that exists, the more capital there is to wear out in a given time period*).

With growth in the size of the labor force (*due to population growth and increasing labor-force participation rates*), additional investment is also necessary to maintain the capital-labor ratio (K/L) at a particular level. Thus, the level of investment must exceed rate of depreciation by an amount equal to the growth-rate '**n**' in the labor-force:

$$I_{required} = \mathbf{(d + n)}K$$

or if we divide both sides by 'L'

$i_{required} = \mathbf{(d + n)k}$ – *required investment per worker*

Investment is possible only if a given country makes resources available for this accumulation of capital. These resources, known as savings, represent those goods produced in the current time period not devoted to final private or public consumption. In a closed economy, we could write the following:

Savings: $S = \mathbf{Y^*} - C - G$

$S = \mathbf{sY^*}$

where '**s**' represents the proportion of output (resources) not devoted to private consumption 'C' or public (Government) consumption 'G'. With efficient financial and capital markets, these savings could then be made available for investment in new capital:

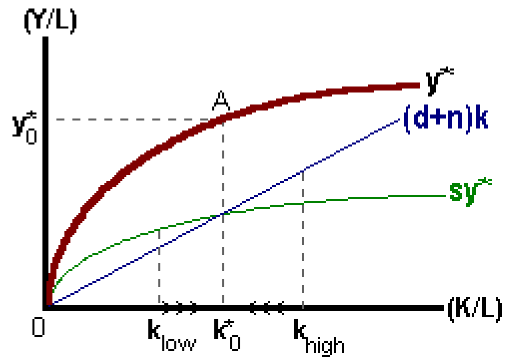
$$\text{Savings} = I_{required}$$

or in per-capita terms:

$$\mathbf{sy^* = (d + n)k}$$

A "steady-state" level of capital is defined as the above equality and is modeled by the intersection of '**sy***' (*per-capita savings as a proportion of per-capita income*) and '**(d + n)k**' in the diagram below:

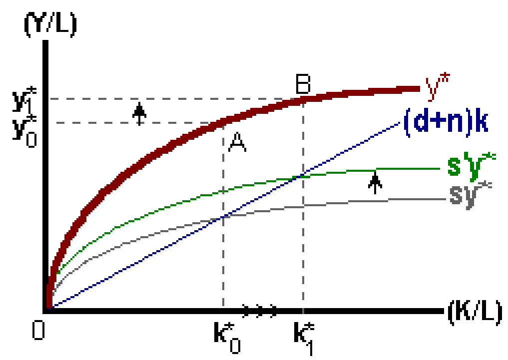
Figure 6, The Steady State



If **sy*** < **(d + n)k**, then savings (*and thus investment*) is below the level necessary to compensate for the rate of depreciation and growth in population. In this case capital is wearing out faster than it can be replaced or labor input is growing faster than capital is added. In this case, the capital-labor ratio is decreasing (Δ**k** < 0). If the opposite is true, the amount of capital available per unit of labor is increasing (Δ**k** >0). This latter situation leads to economic growth as measured by changes in per-capita output (**y*** ↑).

According to this model, an increase in the rate of savings leads to growth in the capital stock, a higher steady-state capital-labor ratio, and therefore a higher Standard of Living. Reductions in population growth rates '**n**' may accomplish the same result.

Figure 7, An Increase in Savings Rates



See: The Digital Economist: http://www.digitaleconomist.com/solow.html
To practice with changes in the parameters of the Solow model.

It is important to note that even though an economy may be in a "steady-state" condition, this does not imply that there is no growth in factor inputs or output. If **k** is constant, this implies that the %ΔK = %ΔL. In addition, given that **y*** is also constant in the steady state, %ΔY* = %ΔL. Thus we can write:

$$\%\Delta Y^* = \%\Delta L = \%\Delta K.$$

or

$$\%\Delta Y^* = \alpha[\%\Delta L] + (1-\alpha)[\%\Delta K].$$

If we observe an economy with a growth rate that exceeds the rate of population growth, we can conclude that this economy is currently below its steady state level of capital per unit or labor or that this growth is due to exogenous changes in technology (i.e., the $\%\Delta\mathbf{A}_t > 0$).

Be sure that you understand the following concepts and terms:

- Standard of Living
- Stock and Flow Variables
- Aggregate Production
- Wealth
- Rate of Time Preference
- Diminishing Marginal Productivity
- Constant Returns to Scale
- Profit Maximizing Behavior
- Economic Growth
- Cobb-Douglas Production Function
- Savings Rates
- Rate of Depreciation
- Population Growth Rate
- Nominal Wage Rate
- Real Wage Rate

See: The Commanding Heights for additional review:
1) Growth and GDP
http://www.pbs.org/wgbh/commandingheights/lo/educators/ed_u1_gdp_exercise.html

2) Per-capita Income
http://www.pbs.org/wgbh/commandingheights/lo/educators/ed_u1_percapita_exercise.html

3) Growth and Policy
http://www.pbs.org/wgbh/commandingheights/lo/educators/ed_u1_policy_exercise.html

and *The Digital Economist:* http://www.digitaleconomist.com/cap_4020.html